Engineering Students Understanding Mathematics (ESUM): an innovative teaching approach with integrated research

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Abstract

Engineering Students’ Understanding of Mathematics (ESUM) is a developmental research project involving an innovation in teaching designed to achieve a higher level of engagement and conceptual understanding in mathematics for first year engineering students. The innovation has included small group work, a group project, inquiry-based questions and tasks and use of GeoGebra. Research has both studied the teaching process and its outcomes and contributed to the impact of the project. Increased student engagement and understanding have been evidenced through observations of activity, focus groups and interviews and a range of assessments. We look critically at approaches to teaching and a range of issues related to achieving teaching goals.

Keywords: engineering students learning mathematics, engagement, understanding, approaches to teaching, use of resources.

Background

The School of Mathematics at Loughborough University (LU), led by its Mathematics Education Centre (MEC), teaches mathematics to students of engineering within LU. In collaboration with the Faculty of Engineering and the Engineering CETL, we set out to enhance participation and understanding of first year Materials Engineering students in mathematics through an inquiry-based approach involving group engagement with mathematical software and to study outcomes. The project was intended to pioneer an innovative methodology towards developing teaching and the design of teaching with a particular cohort of students, but its focus was generic; we expected to use our findings to influence design of teaching broadly within LU and beyond.

Students in Materials Engineering have a wide variation in prior qualifications which makes teaching much more difficult (some are registered for B.Eng or M.Eng degree and others for B.Sc degrees having different pre-requisites). The 2010-11 cohort of students (48) had achieved mathematics mainly at A level in grades A, B or C. Two students had only GCSE mathematics, two only AS level mathematics and two the baccalaureate.

The approach taken to teaching these students could be described as inquiry-based, with integral use of a computer package GeoGebra (linking graphical and algebraic representations). Students were grouped in 3s or 4s for activity in tutorials, and each group had to complete a group project which was assessed. Assessment changed to accommodate this project. A former eight CAA tests, each worth 5% (i.e.40% in total), were reduced to four (i.e. 20%), thus allowing 20% for the

1 Developmental research is research which actually influences the developmental process as well as charting the development (Jaworski, 2003).

2 GeoGebra is free software which allows both algebraic and graphical representations of a function to be displayed side by side on a screen. Graphs can be modified by dragging points and lines on the screen and observing how this affects dependent elements. http://www.geogebra.org/cms/
project. An exam worth 60% was continued from previous years. The small group work, project, and use of GeoGebra, together with specially designed inquiry-based questions and tasks, were intended to increase student engagement and conceptual understanding in mathematics. We expected to design these approaches, to use them in real teaching-learning situations in the university context, and to study the entire developmental process.

Rationale

The project aimed to develop/improve participation and mathematical understanding of first year engineering students through inquiry-based innovation in practice and to study outcomes. It was intended to:

- work closely with engineering subject areas and develop links between mathematics teachers and engineers.
- use a developmental research approach and evaluate its contribution to the teaching process and to students’ participation and understanding.
- evaluate the contribution made by mathematical software and a pedagogic approach of inquiry-based activity, grouping students for activity and assessment.
- disseminate outcomes to colleagues in both mathematics and engineering both in and beyond LU.

A starting point was our recognition that engineering students’ engagement with mathematics is often of a rather instrumental nature – rule following without deeper conceptual understandings. We were also aware of the many other challenges associated with teaching mathematics to engineers – reported ill-preparedness, progression and retention issues. Published research both provides evidence of this and suggests reasons for it (Skemp, 1976; Hawkes and Savage, 2000; Artigue, Batanero and Kent, 2007). Research has shown that approaches to teaching mathematics using technology and collaborative activity can promote better engagement and understanding, and thereby improve progression and retention (e.g., King, Hillel and Artigue, 2001; Trouche Drijvers, 2010). An inquiry-based approach to learning engages students in collaborative exploratory activity through which they ask their own questions and take up their own lines of inquiry and hence develop a more conceptual understanding of mathematics (Jaworski, 1994; 2006; Wells, 1991). Teachers’ inquiry into the teaching-learning process leads to a more knowledgeable view of mathematics learning and ways of promoting conceptual learning. Developmental research, involving closely-linked elements of both research and development, underpinned ESUM activity to ensure that our inquiry into the teaching-learning process both promoted students’ engagement through inquiry and monitored its nature and progress (Jaworski, 2003, 2008). In this project we have used theory and approaches developed in projects involving mathematical learning at pre-university levels adapted to the higher education context, and consider how this context limits or extends what is possible. Theories of instrumentation (e.g. Trouche & Drijvers, 2010) and documentational genesis (e.g. Gueudet & Trouche, in press) help to explain the ways in which the resources used by the teacher in the processes of teaching and by students in their processes of learning, together with their schemes of utilisation, facilitate the desired engagement and understanding. We shall elaborate on these theoretical perspectives in papers published from this work.

The Approach

The overall design of the ESUM project

The project was designed in four phases, each of 3 months: 1) design 2) activity; 3) analysis; and 4) dissemination.

1. The design of teaching and associated research: the team worked collaboratively on this design.
2. The actual teaching, by one member of the team, with an inquiry cycle looking critically into how design is interpreted and learning achieved (Jaworski, 2008). Data were collected, by the research officer, from student participation and achievement.

3. Analysis of data and synthesis of findings, by the research officer guided by the team as a whole.

4. Writing reports/papers and other dissemination of both a professional and a research nature.

The innovative nature of the project, stimulating inquiry, included a number of elements:

- using computer software (GeoGebra) as a tool to promote mathematical conceptualisation;
- organising students into groups to work together on mathematical tasks using software;
- an assessed group project in which members of group worked on an exploratory activity and presented a group report and poster;
- specially designed inquiry-based questions presented in lectures and tutorials to promote engagement with mathematics and conceptual thinking.

A research officer worked closely with the research team to collect data from teaching events, conduct interviews with students, and analyse data along with the team. The research was developmental, contributing to development as well as providing evidence of its achievements and associated issues. Ongoing feedback from research informed interpretation of the design of teaching and enabled local adjustments to be made relative to student participation and achievement. We have measured effectiveness through conversations/focus groups with the students within which they talk about their experience, and whether the use of the software/group problem solving has changed their attitudes to mathematics.

The overall design has been achieved collaboratively with colleagues from engineering departments and the Engineering Subject Centre. The research officer has worked closely with the team member doing the teaching to enable linking teaching and research. The team as a whole has engaged in processes of design, analysis and synthesis, with the research officer carrying out the research process as agreed.

A note is necessary here. During the activity phase of the project, the same team was successful in its bid for the second call for STEM projects. Our proposal was to extend the first project in order to achieve some of its aims more thoroughly, basically to achieve a more rigorous research approach and further dissemination. This included a literature review, more time for analysis of data and for dissemination activity. Thus, what we are able to report in this first case study will be extended in our case study from the second project in which analysis is still ongoing and some dissemination events still to be organised.

**Action within the project**

In the design phase we achieved the following:

- Money from another source paid for a BSc mathematics graduate to work with two team members to design the group project in three versions. The three versions allowed for variation across the cohort with scope for groups to learn from each other’s work.
- The team met on several occasions to discuss the teaching approach and integration of the new elements into an established teaching format (two lectures and one tutorial per week for 13 weeks).
- An advisory group was set up and had a meeting to consider aspects of the project at its planning stage. The advisory group included the Director of the EngCETL at LU and a member of the Materials Engineering Department. From the beginning, we have communicated closely with our Royal Academy of Engineering advisor, Ivan Moore, and he has attended some of our meetings.
- Two team members, with a research student, explored sources of inquiry-based questions relating to parts of the module. Specific questions were incorporated into lecture and tutorial material and the teaching-learning interface. Further questions were designed as appropriate.
- Lectures and tutorials were designed, building on previous years’ material and incorporating the new elements. Each lecture included a PowerPoint presentation as well
as supplementary examples and tasks; each tutorial had a problem sheet on which students would focus in groups using GeoGebra.

The activity phase was the main teaching-learning part of the project in which design was put into practice using the various elements of the innovation and implementing them in the practical setting. 48 students were registered. In the first week of teaching-learning, students were grouped in threes or fours. This was done by the lecturer (the team member doing the actual teaching) according to the programmes on which the students were registered to enable them more easily to meet together outside lecture/tutorial time. Students were expected to work with members of their group in tutorials and on a group project which would involve some work in their own study time.

Lectures involved several elements: presentation of material using PowerPoint; mathematical examples, often presented in real time on an OHP; questions to students; tasks for students to undertake during lecture time. The lecturer made considerable effort to communicate directly with students during the lecture and to learn their names. Students were encouraged to respond to questions and to ask questions themselves. The lecturer’s questions included a variety of pre-designed open or inquiry-based questions, and many questions of a more closed nature to encourage response and monitor engagement and understanding. In tutorials, held in a computer laboratory, students sat in their groups around sets of four computer tables in a square arrangement. Each student was asked to load GeoGebra onto their own computer. Members of each group were asked to work together on a set of tasks on a specially-prepared task sheet linked to the material of current lectures. The lecturer and a graduate assistant circulated, talking with groups about their work. The style of interaction was, deliberately, one of reflecting questions back to students, getting them to engage with concepts and only providing answers or explanations where this seemed most appropriate. We encouraged their own articulation of mathematical ideas and observations, and use of canonical language where relevant.

During the second week, students were given details of the group project and its requirements. Three tutorials (in weeks 3, 6 and 9) were devoted to the project, with opportunity to ask questions and seek help, and groups were expected to spend further time organised by themselves. The lecturer encouraged students to contact her by email if they had questions, issues or problems, and met certain students face to face on occasion outside lecture hours.

The research officer observed all lectures and tutorials, keeping field notes and audio-recording lectures. In collaboration with the lecturer, she produced and distributed two questionnaires to students, the first, in week 1, for baseline information on students, and the second, in week 6 for information to date on students’ perceptions of the module. The research officer discussed with the lecturer ongoing aspects of the teaching and issues arising, and such discussions fed back into lectures or tutorials as part of the overall developmental process. For example, feedback from analysis of the second questionnaire was given to students in week 7. Throughout the planning and activity stages we collected data to allow us to evaluate whether student engagement with mathematics increased (over previous years) and whether there was a corresponding improvement in assessment results (see table below).

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<th>Start of module</th>
<th>During module</th>
<th>End of module</th>
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<tbody>
<tr>
<td>Reflection on previous two years</td>
<td>Lectures – reflection, observation, attendance</td>
<td>Reflection on module engagement</td>
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<tr>
<td>Assessment results of previous two years – CAA, exam</td>
<td>Normal tutorials with new questions – reflection, observation, discussion with students, attendance</td>
<td>Assessment – CAA, exam, project</td>
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<tr>
<td>Attendance of previous two years</td>
<td>Normal tutorials with old and new questions – reflection, observation, discussion with students, attendance</td>
<td>Project evaluation</td>
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Table showing modes of data collection across the module
Data were analysed to characterise teaching, use of software, student participation and student achievement. This analysis has addressed research questions related to the main aims including the following:

- How are students’ participation and achievement related to innovative modes of teaching and assessment?
- What issues arise in the developmental process and how can these inform teaching and assessment more widely?
- In what ways do communications between mathematics teachers and engineers contribute to wider understandings of mathematics teaching and learning of engineering students?

Specific questions relating to evaluation have been:

- Have the innovations in teaching led to an increase in student engagement with mathematics, confidence with mathematics, enjoyment of mathematics, an increase in attendance, better results in assessment?
- What has been the role of questions in lectures, questions in tutorials, lecture slides, project questions, Geogebra, working in a small group, more challenging CAA tests?
- What changes would be made next year?

**Assessment**

Impact on assessment has been considerable. At the design stage we modified the assessment of the module as shown in the table below. It is important to say here that the module took place over two semesters. The innovation was part of the first semester only, focusing on pre-calculus (functions, matrices and complex numbers). In the second semester another lecturer taught calculus. This was not part of the ESUM study. However, the module was assessed as a whole as the table shows.

<table>
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<th>Original assessment</th>
<th>Revised assessment</th>
<th>Comments</th>
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<td>8 CAA tests (each 5%). 4 in the first semester covering functions (polynomial, rational, trigonometric, exp and log) complex numbers and matrices 4 in the second semester.</td>
<td>4 CAA (5% each) tests 2 in the first semester covering functions (polynomial, rational, trigonometric, exp and log) complex numbers and matrices 2 in the second semester.</td>
<td>The 2 CAA tests in the first semester included most of the questions from the original 4 tests. Experience had shown that most students could cover this material in less time than was originally provided.</td>
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One 3-hour final examination (choice of 6 from 8 questions in 3 hours), half of it devoted to the first semester work and half to the second semester work.

60%

We kept to the same style and format of examination, as changing this would have had implications beyond current design.
**Project reports:** For 13 groups, 11 submitted a complete report in which they had obtained feedback from another group and had commented on this feedback. One group failed to meet the final submission deadline despite clear instructions and reminder. The average score was 61.3% (excluding zeros).

**Posters:** For 13 groups, 12 posters were submitted (only one group did not produce a poster – they had 75% for their project report). Out of 48, 36 students received non-zero marks for the poster (average 6/10). In 5 groups – some students participated; in 7-groups all students participated.

**Attendance:** In the first semester attendance increased and may be an indication of increased engagement. (2006/07-48%, 07/08 – 54%, 08/09 – 47%, 09/10 – 58%, 10/11 – 66%).

**Failure rate:** Three students failed the module. One of these was one of two students who were admitted to the module with only GCSE mathematics (Grade C). The work was very difficult for both of these students, although one of them passed comfortably. The third failing student achieved low marks throughout and had problems with attendance.

**Evaluation**

The project is both developmental and research led. We have evaluated its success in terms of the interpretation of design of teaching into actual teaching and the ways in which students participate in and learn from their activity in teaching-learning sessions. Design, teaching and research have been interwoven within an analytical frame which looks critically at what is being achieved in parallel with teaching-learning activity (Jaworski, 2003; 2008). We have collected data as indicated above and analysed the data to address our main research aims as set out above. Analysis has taken a form relevant to the kind of data collected. Thus survey data have been analysed through simple counts and percentages; qualitative data from interviews, focus groups, reflective writing and observation notes have been analysed through a grounded approach which is ongoing. Detailed findings from qualitative analysis will be presented as part of our second case study in which we will address development of teaching in relation to theoretical perspectives leading to a theoretical rationale for the new knowledge of teaching which is a result of our study.

Early analyses of observation and reflective data suggest that lectures, including Powerpoint presentations, examples worked in real time using the OHP, questions of various kinds, including inquiry-based questions, and small tasks for students resulted in an increased level of interaction compared with previous years. This involved students responding to questions, asking questions themselves and some discussion between students. Tutorials involved a buzz of activity and discussion in group work, either on set inquiry-based tasks relating to the week’s topic, or on project activity. Each group worked at its own set of computer workstations talking together, working with GeoGebra and discussing with the lecturer or graduate assistant. There were some instances of students using email or Facebook, but not noticeably many. There was discussion off task, but this did not predominate. In the main students focused on the mathematical tasks and the lecturer and assistant encouraged a conceptual engagement. When the cohort was taken over in the second semester by the second lecturer, this colleague commented spontaneously on the level of communication that was instantly established which allowed her to work more interactively with these students than with previous cohorts.

Project reports indicate students’ perceptions of their learning through the projects and in relation to GeoGebra. Relevant statements from different groups are

“We have also learned a lot about manipulating functions from trying to get the best fit line to fit the data on the graphs. GeoGebra was very helpful for this.” [C]

“...when somebody has had any difficulties others in the group have stepped forward to help them.” [A]

“This project has been very useful to all members of the group in understanding functions; one of the main observations was how much the line of a function could be changed by changing [parameters], to in fact produce a line that is completely different from the original, all functions have a basic shape but the gradient, size and even direction can be changed.” [L]
“……Seeing these practical uses for maths has made us see that it is far more useful than it seems when just working with the theory of maths in lecture rooms.” [C]

“We think that people, including us, may think that they understand functions but it is always very useful to see a graphical form of the function and this does give a better understanding of what happens as \((x,y)\) values increase.” [A]

“We feel being able to explore functions as a group has helped our learning about functions as we can discover together and ask each other questions about how they work and what they can be useful and where one student questions something, the whole group benefits from their answer.” [I]

We expect to include quotations from students from interviews and focus groups in our second case study. However, initial analyses show that the approach used has been demanding on students in terms of their participation. Those who have gained the most have attended lectures and tutorials, taken the group activity seriously and participated well alongside their peers, and engaged seriously with mathematical ideas. There is some correlation between marks and attendance, with students who have scored the lower marks having a poorer attendance record than many of the others. A few students have indicated that they would have preferred not to have to work in groups, and to have more CAA tests rather than the project work. These students are very much in the minority. Most of the students have appreciated the group work and are aware of their deeper understanding of functional concepts as a result of engaging with the project and with GeoGebra. Some students who have come to the module with good A level grades have suggested that the work on functions has repeated much that they had done previously at A level. They would have liked to spend more time on matrices and complex numbers which they had not done before.

These findings together with results from survey analysis and module assessment suggest that areas of achievement in the project are

1. Greater participation (mathematical engagement) by students in lectures possibly responding to greater effort (than in previous years) by the lecturer to include students through frequent questioning and inviting students to respond, comment and ask their own questions.
2. Higher student attendance in lectures and tutorials than in previous cohorts.
3. Pleasing (to the lecturer) response by students to group work in tutorials and to project work. Enhanced engagement was commented on particularly by the graduate student who has helped in tutorials over two years. VERY pleasing participation in design of a poster.
4. A good average mark for the projects.
5. CAA scores at about the same level as for previous cohorts, despite each CAA test having almost twice the number of questions for the same amount of time.
6. A considerably higher exam average on the module as a whole.

Discussion, Summary

Key outcomes are outlined above. Analysis to date has supported the view that students have shown greater participation and evidence of greater conceptual understanding. Remarks from some students suggest that the work on functions early in the module may seem to be a repetition of previous work and therefore not requiring their focused attention. This must be attended to in future work. The use of GeoGebra and small group work in tutorials and on the project have all been beneficial. Small details with respect to all of these can be adjusted in relation to specific feedback from students and observations by lecturer and researcher.

The following are areas where different approaches may be designed for the coming year:

1. It may be worthwhile to start with a topic on matrices which most students have not met before. They will not therefore make the mistake of thinking that the course material is just a repetition of previous work.
2. Because most students have studied functions at A level, albeit mainly with a rather instrumental approach, basic bookwork on functions will be reduced and replaced with more searching inquiry-based questions. Lectures will focus on solution of inquiry-based problems and tutorials will present students with inquiry-based tasks. Students needing a more definitional approach will be directed to the HELM books which will still be provided as a resource.

3. Due to the apparent success of greater interaction in lectures, the lecturer will continue to use and encourage questions in lectures and possibly increase opportunity for such interaction.

4. The project will be continued, with minor adjustments. The timing of the project will be adjusted to fit rearrangement of topics.

5. Small group work will be continued. Selection of groups will be discussed with the Materials Engineering department to avoid students being in the same group for too many of their modules.

The project will definitely have enhanced professional practice where the current lecturer is concerned. Details of professional enhancement will be left until further analysis is complete and reported in the second case study.

Further Development

We have mentioned above particular areas in which changes may be made to the module next year as a result of current learning. These will lead to establishing and sustaining the new approach as the way this module will continue to be taught. Dissemination events will be planned to communicate aspects of this approach to mathematics lecturers in both engineering and mathematics, so that other modules may take advantage of what we have learned here. Opportunities for communication with lecturers outside LU will be taken through local opportunities and at conferences national and international.

Communication has already been made with a group of researchers in Dublin and joint research will take place in 2011-12 on the further use of inquiry-based questions to promote conceptual understanding. We plan to develop an instrument to measure increased understanding of the students involved in three institutions.

These and other dissemination issues and practices will be discussed in our second case study.

References


Skemp, R. (1976). Relational understanding and instrumental understanding. Mathematics Teaching, 77


Further Reading / Bibliography

The following papers are relevant. The first two relate to the ESUM project. Readers who would like to look at Powerpoint slides, inquiry-based questions and tasks and tutorial sheets, or copies of questionnaires may contact Barbara Jaworski on b.jaworski@lboro.ac.uk

Jaworski, B. (2010). Engineering students understanding mathematics ESUM. In BSRLM Proceedings: Vol 30 No 3 Newcastle University, Nov 2010


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