The Mathematics of Engineering Surveying (1)

Scenario

As a new graduate you have gained employment as a graduate engineer working for a major contractor that employs 2000 staff and has an annual turnover of £600m. As part of your initial training period the company has placed you in their engineering surveying department for a six-month period to gain experience of all aspects of engineering surveying. One of your first tasks is to work with a senior engineering surveyor to establish a framework of control survey points for a new £12m highway development consisting of a two mile by-pass around a small rural village that, for many years, has been blighted by heavy traffic passing through its narrow main street.

In this exercise you will carry out the geometric calculations that would enable you to determine the precise coordinates of the control survey points. These calculations are based on site measurements obtained through a process known as *traversing.*

Importance of Exemplar in Real Life

In establishing the line of a new highway the engineering surveyor will initially establish a control network which is the framework of survey points or 'stations' for which the co-ordinates are precisely determined. The points are considered definitive and subsequent survey work, such as then establishing chainage points along the road centre-line, are related to them.

Physically, these stations will consist of, for example, short nails embedded in a road surface or nails cast into concrete-filled steel tubes driven into the ground. In all cases the stations must be rigid and not prone to disturbance or movement. Their exact location will depend on the purpose of establishing the network and, in the case of a major highway, bends in the road and the need to be able to sight between stations may dictate the pattern of stations.

Care needs to be taken over the provision of control so that it is precise, reliable and complete as it will be needed for all related and dependant survey work. Other survey works that may use the control will usually be less precise but of greater quantity. Without an accurate control network it would be impossible to ensure the correct alignment of the highway or the accurate positioning of ancillary works such as service stations and junctions with other roads.

Apart from the specific example indicated in this exercise, other examples include setting-out for earthworks on a construction site, detail surveys of a green field site or of an as-built development and monitoring many points on a structure suspected of undergoing deformation.

The practice of using a control framework as a basis for further survey operations is often called 'working from the whole to the part'. If it becomes necessary to work outside the control framework then it must be extended to cover the increased area of operations. Failure to do so will degrade the accuracy of later survey work even if the quality of survey observations is maintained.

Traversing is one of the simplest and most popular methods of establishing control networks for all types of major projects. In underground mining it is the only method of control applicable because of the linear nature of tunnels whilst in general civil engineering it lends itself ideally to control surveys where only a few points surrounding the site are required. Traverse networks have the advantages that little reconnaissance is required so planning the task is simple.

Examples of Civil Engineering schemes that would require the establishment of an accurate control network before any construction commences are shown in figures 1 to 4

Figure 1: *Construction of a major dam*

Figure 2 *Construction of a highway*

Figure 3: *Tunnel Construction*

Figure 4 *Airport construction*

Background Theory

Types of Traverse

Essentially there are two types of traverse: (a) closed and (b) open. These are illustrated in figures 5(a) and 5(b) respectively. In both cases the traverse consists of a network of stations, marked as A,B,C… and traverse lines joining the stations together. In this explanation we will only be concerned with traverses in the horizontal plane therefore all references to angles in this text relate to angles measured in the horizontal plane. The coordinates of the first traverse point (A) must be known as well as the bearing of station B from station A. This may have been separately established by reference to the National Grid – the purpose of the traverse is to establish the National Grid coordinates of all other stations within the traverse.

If, as in figure 4(a) the lines form a closed polygon then the traverse is 'closed', if not it is 'open'. The choice between which type of traverse to use depends on its purpose such that, for a long highway, a long unclosed traverse is likely to be used although, if available within the locality of the highway, intermediate and end stations should, en-route, be tied-in to existing Ordnance Survey Triangulation Stations whose National Grid coordinates will be known. This will effectively "close" the traverse.

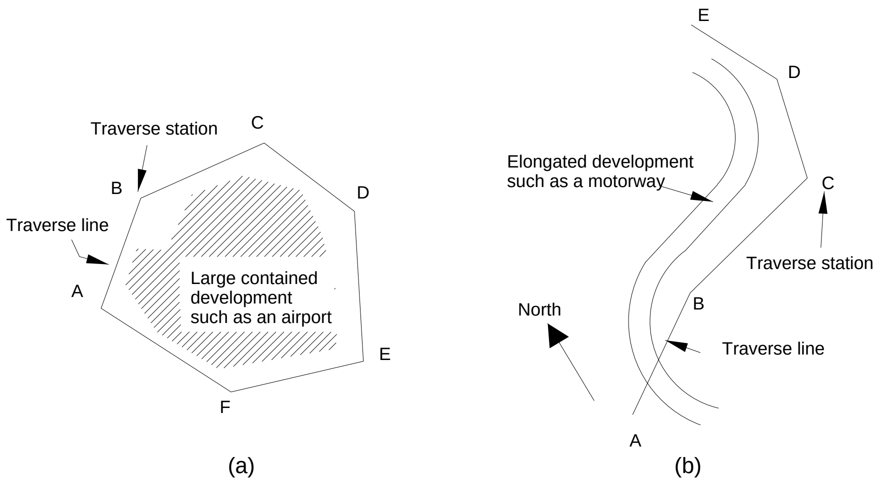

Figure 5: Closed and Unclosed Traverses

Traverse Measurements

In the field, the Surveyor will measure:

(a) *The Length of each Traverse Line*

Traditionally, this was done by using a steel band and applying a series of corrections to allow for the sagging of the tape, temperature effects, and so on. Today it will be done electronically with the Surveyor using a highly sophisticated *Total Station* set of equipment that can measure both distances and angular measurements to a high level of accuracy.

(b) *The Bearing of each Line*

The bearing of a line is its orientation with respect to grid North. This may be found for an initial line by starting at a station with known coordinates and orienting the Total Station with respect to another point with known National Grid coordinates such as a church spire. The Surveyor will locate the Total Station accurately over each traverse station in turn and will measure the horizontal angles as indicated in figure 6 below. These measurements are to a high level of accuracy and several readings will be taken to minimise the possibilities of measurement errors. The angles are measured in degrees, minutes and seconds. (1 degree = 60 minutes; 1 minute = 60 seconds)

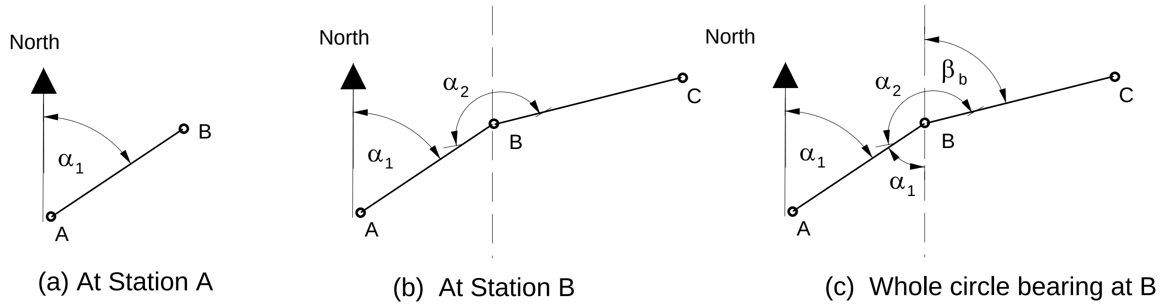

Figure 6: Angular Measurements and Bearings

In figure 6(a) the coordinates of station A will be known and the distance AB will be measured together with the bearing which is referred to as the *whole circle bearing* of B from A and is measured clockwise from the North.

Figure 6(b) indicates that the Surveyor will set up the instrument at B and measure the *clockwise* angle ABC, α_2, the distance BC and, as a check on previous distance measurement AB, the reverse distance measurement BA; the two of course should be the same.

It can be seen from the geometry of figure 6(c) that the *whole circle bearing,* β_B*,* of the line BC is given by:

$$\beta_B = \alpha_1 + \alpha_2 - 180^O$$

Progressing around the traverse in this way the Surveyor will establish the distances between adjacent stations and the *whole circle bearing* from each station to the next. For the purpose of this exercise the learner should draw out the geometry of each station and ensure that the correct geometrical relationships are established.

Traverse Calculations

The purpose of the traverse is to establish the co-ordinates of each station starting from a single station of known co-ordinate which we have taken to be station A. Figure 7(a) shows the geometrical relationship of A to B.

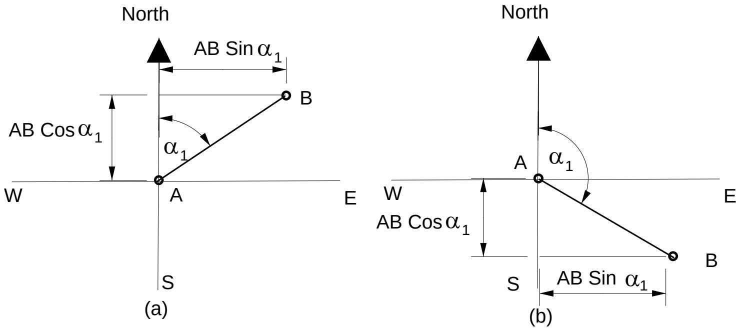

Figure 7: Difference Eastings & Difference Northings

The eastward component of the distance from A to B is known as the *Difference Easting* whilst the northward component of the distance is known as the *Difference Northing*. Hence from the geometry of figure 7(a):

Difference Easting of B from A = $ABSin\alpha_1$
Difference Northing of B from A = $ABCos\alpha_1$

Hence if the coordinates of station A are known then the coordinates of station B are given by:

Easting (B) = Easting (A) + $ABSin\alpha_1$
Northing (B) = Northing (A) + $ABCos\alpha_1$

This process can be repeated from station to station to calculate the coordinates of each station in turn. However care needs to be taken to ensure that the bearing is being used correctly in the calculation. For example, in figure 7(b) station B lies to the east and south of A so the Difference Easting and the Difference Northing are still based on the bearing α_1. Similar considerations would apply if B were to be located to the west of A, either to the north or south. In this case $Sin\alpha_1$ is negative so the Difference Easting is also negative, reflecting the fact that the easting of B will be less than that of A. Again, $Cos\alpha_1$ is negative if B is to the south of A. An experienced engineering surveyor would be able to automatically cope with these different situations but for the benefit of this exercise the learner, as previously noted, should draw out the geometry of each station and ensure that the correct geometrical relationship is established.

Errors

Even with the most accurate of angular and distance measurements in any traverse over a long distance errors will creep in. Large errors are not acceptable but making small adjustments in the calculations can compensate for small errors. This will not be covered in any detail here but it should be noted that in a closed traverse, such as shown in figure 5(a), a simple accuracy check can be applied by noting that geometrically the sum of all the internal angles must equal [$(2n\text{-}4)x90^o$] where *n* is the number of sides of the traverse. Also, if the traverse closes back on itself the sum of all the calculated Differences Easting must sum to zero and the sum of all the Differences Northing must sum to zero (taking account of the signs of these calculated values). Similar but different checks can be applied to unclosed traverses where they can be tied in to points of known coordinates.

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Questions

Calculate the coordinates of the traverse points for a section of a control network established prior to the construction of the new highway, data as given in the table below.

Example Data (1):
The coordinates of station A in the local system are defined to be Easting 1000.000 metres and Northing 2000.00 metres. The coordinates of F have been previously calculated by other surveys to be Easting 1558.27 metres and Northing 2253.93 metres. The known initial bearing from A to B is 45° 10’ 10’’.

Station	Measured Distance to next station	Measured clockwise angle			Measured Clockwise angle	Easting	Northing
	(metres)	(degrees	mins	seconds)	(degrees)	(metres)	(metres)
A	110.45	45	10	10	* 45.1694	1000.00	2000.00
B	121.33	185	30	30	185.5083		
C	99.86	196	10	24	196.1733		
D	169.27	200	10	25	200.1736		
E	135.26	160	45	45	160.7625		
F							

* This is not an angle observed at A but the known initial bearing from A to B.

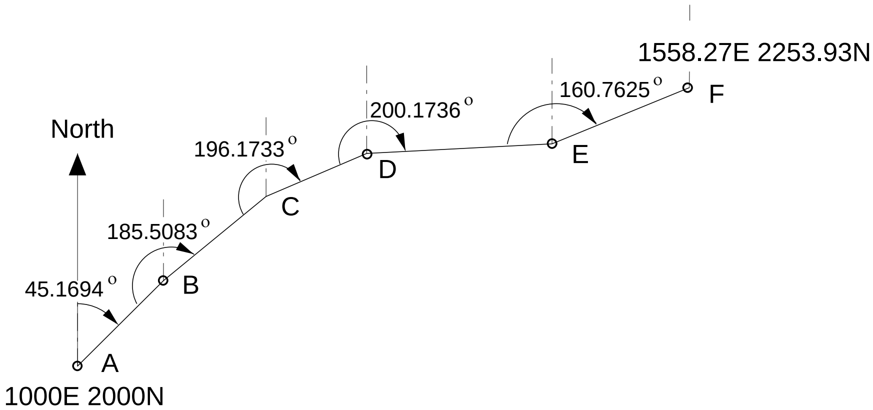

Figure 8: Visualisation of data for Question 1

Example Data (2):
The coordinates of station A in the local system are defined to be Easting 1306.12 metres and Northing 1888.85 metres. The coordinates of F have been previously calculated by other surveys to be Easting 1397.90metres and Northing 2185.14 metres. The known initial bearing from A to B is 30° 10’ 0’’.

Station	Distance to next station	Measured clockwise angle			Measured clockwise angle	Easting	Northing
	(metres)	(degrees	mins	seconds)	(degrees)	(metres)	(metres)
A	98.00	30	10	0	*30.1667	1306.12	1888.85
B	122.35	270	25	35	270.4264		
C	125.46	95	8	15	95.1375		
D	135.67	89	18	22	89.3061		
E	97.36	220	5	55	220.0986		
F							

* This is not an angle observed at A but the known initial bearing from A to B.

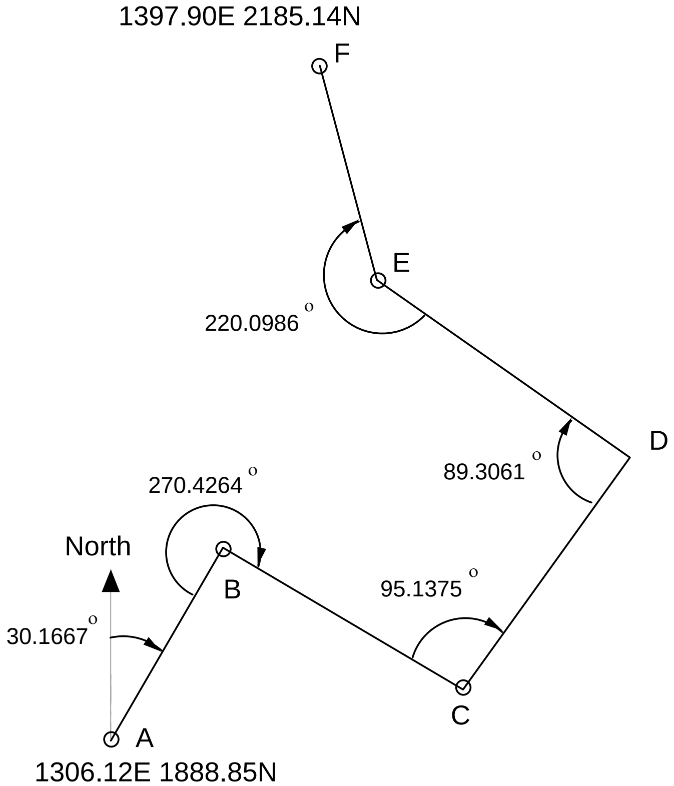

Figure 9: Visualisation of data for Question 2

Where to find more

1. Schofield W, Breach M, *Engineering Surveying*, 6th edn, Oxford: Butterworth-Heinemann
2. Bird J, *Engineering Mathematics,* 5th edn, Elsevier, 2007 (ISBN 978-07506-8555-9)

The Mathematics of Engineering Surveying (1)

INFORMATION FOR TEACHERS

CONSTRUCTION AND THE BUILT ENVIRONMENT DIPLOMA

Teachers will need to understand and explain the theory outlined above and have knowledge of:
- Some terminology relating to engineering surveying
- Geometry and trigonometry

Topics covered from Mathematics for Engineers

Topic 1 Mathematical Models in Engineering
Topic 5 Geometry

Learning Outcomes

- LO 01: Understand the idea of mathematical modelling
- LO 05: Know how 2-D and 3-D coordinate geometry is used to describe lines, planes and conic sections within engineering design and analysis
- LO 09: Construct rigorous mathematical arguments and proofs in engineering context
- LO 10: Comprehend translations of common realistic engineering contexts into mathematics

Assessment Criteria

- AC 1.1: State assumptions made in establishing a specific mathematical model
- AC 1.2: Describe and use the modelling cycle
- AC 5.1: Use equations of straight lines, circles, conic sections, and planes
- AC 5.2: Calculate distances
- AC 9.1: Use precise statements, logical deduction and inference
- AC 9.2: Manipulate mathematical expressions
- AC 9.3: Construct extended arguments to handle substantial problems
- AC 10.1: Read critically and comprehend longer mathematical arguments or examples of applications

Links to other units of the Advanced Diploma in Construction & The Built Environment

Unit 2 Site Surveying
Unit 3 Civil Engineering Construction
Unit 6 Setting Out Processes

Solution to the Questions

Example Data (1):

Station	Reduced bearing	Reduced bearing	Difference Easting	Difference Northing	Easting	Northing
	(degrees)	(radians)	(metres)	(metres)	(metres)	(metres)
A	45.1694	0.78835	78.33	77.87	**1000.00**	**2000.00**
B	50.6777	0.88449	93.86	76.88	1078.33	2077.87
C	66.8510	1.16677	91.82	39.26	1172.19	2154.75
D	87.0246	1.51887	169.04	8.79	1264.01	2194.01
E	67.7871	1.18311	125.22	51.13	1433.05	2202.80
F					1558.27	2253.93
True F					1558.27 (check)	2253.93 (check)

Example Data (2):

Station	Reduced bearing	Reduced bearing	Difference Easting	Difference Northing	Easting	Northing
	(degrees)	**(radians)**	**(metres)**	**(metres)**	**(metres)**	**(metres)**
A	30.1667	0.52651	49.25	84.73	**1306.12**	**1888.85**
B	120.5931	2.10475	105.32	-62.27	1355.37	1973.58
C	35.7306	0.62362	73.27	101.84	1460.69	1911.31
D	-54.9633	-0.95929	-111.08	77.89	1533.96	2013.15
E	-14.8647	-0.25944	-24.98	94.10	1422.88	2091.04
F					1397.90	2185.14
True F					1397.90 (check)	2185.14 (check)

Similar exercises can be developed to that indicated above. Learners should be encouraged to sketch the geometry of the traverse on graph paper to check the accuracy of their calculations.

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