

INTRODUCTION

JCB construction machinery is the workhorse of many construction sites around the world. JCB is the world's 3rd largest producer of construction machinery. The majority of the range of equipment is powered by the JCB Dieselmox engine – so-called after a modified version of the mass-produced digger engine that was used to power the JCB Dieselmox LSR car to a world land speed record of 350mph in August 2006.



CALCULATION OF POWER

The JCB Dieselmox engine series is a 4 stroke, 4 cylinder range, with power outputs ranging from 63 kW to 120 kW. Each cylinder has a 103 mm bore (diameter of the cylinder) and a 132 mm stroke (maximum travel of each piston). The volume of an engine is defined by the total swept volume of all cylinders. The swept volume of an engine can be calculated as:

$$\begin{aligned} V_s &= n_{cyl} \cdot \frac{\pi \cdot bore^2}{4} \cdot stroke \\ &= 4 \times \frac{\pi \times 103^2}{4} \times 132 \\ &= 4.4 \times 10^6 mm^3 \\ &= 4.4 \times 10^{-3} m^3 \\ &= 4.4l \end{aligned}$$

The power of a 4 stroke engine can be calculated by applying the formula:

$$P = \rho_a \times \frac{V_s}{2} \times S \times \left[\frac{F}{A} \right] \times Q_{lhv} \times \eta \quad \dots (1)$$

where:

P = engine power [W]

ρ_a = air density [kg/m³]

V_s = engine swept volume [m³]

S = engine speed [revs/sec]

$\left[\frac{F}{A} \right]$ = fuel:air ratio [no units]

Q_{lhv} = lower heating value of fuel [J/kg]

η = efficiency [expressed as a decimal]

A watt is defined as a joule per second. We can check this formula via dimensional analysis as follows:

$$W = \frac{kg}{m^3} \cdot m^3 \cdot \frac{1}{sec} \cdot \frac{J}{kg} = \frac{J}{sec}$$

For the naturally aspirated JCB Dieselmox engine (i.e. the air drawn into the engine is that of ambient conditions), we have:

$$\rho_a = 1.2 \text{ kg/m}^3,$$

$$\left[\frac{F}{A} \right] = 0.0382,$$

$$Q_{lhv} = 42.6 \text{ MJ/kg},$$

$$\eta = 0.4.$$

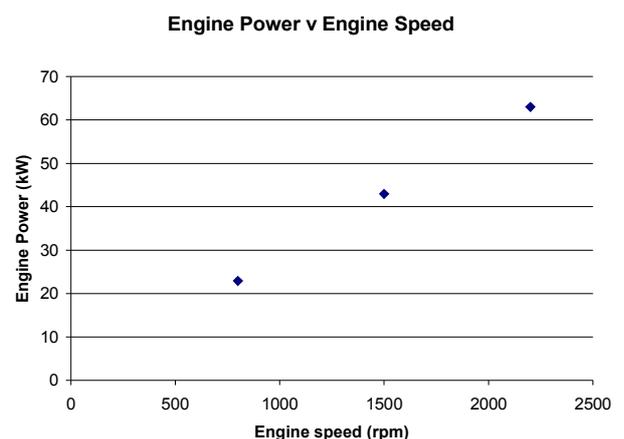
Using equation (1), we can calculate the power P at a speed of 800 rpm assuming all other conditions remain constant as follows:

$$\begin{aligned} P &= 1.2 \times \frac{4.4 \times 10^{-3}}{2} \times \frac{800}{60} \times 0.0382 \times 42.6 \times 10^6 \times 0.4 \\ &\approx 22.91 \text{ kW} \end{aligned}$$

Similarly, the power at 1500 and 2200 rpm can be calculated and tabulated in the following table:

Engine speed, S (rpm)	800	1500	2200
Engine power, P (kW)	22.91	42.96	63.01

Plotting these values, we get the following graph:



We can find the equation of the line and use this to predict the power at other speeds; for example 1000, 1200, 1600, 1800 and 2000 rpm. Using the standard formula for the equation of a line passing through a point, we have

$$y - y_1 = m(x - x_1)$$

Substituting the known values, we get:

$$63 - 22.9 = m(2200 - 800)$$

$$\Rightarrow 40.1 = 1400m$$

$$\Rightarrow m = 0.02864$$

Also, the general equation of a line is

$$y = mx + c$$

From equation (1), it is clear that the engine power is zero when the engine speed is zero. Hence, the Y intercept for such an equation is zero. Therefore, we get the following relationship:

$$\text{Power} = 0.02864 \times \text{speed} \dots (2)$$

Equation (2) is of the form

$$P = m \times S$$

Comparing it with equation (1), we can easily see that:

$$m = \rho_a \times \frac{V_s}{2} \times \left[\frac{F}{A} \right] \times Q_{lhv} \times \eta \dots (3)$$

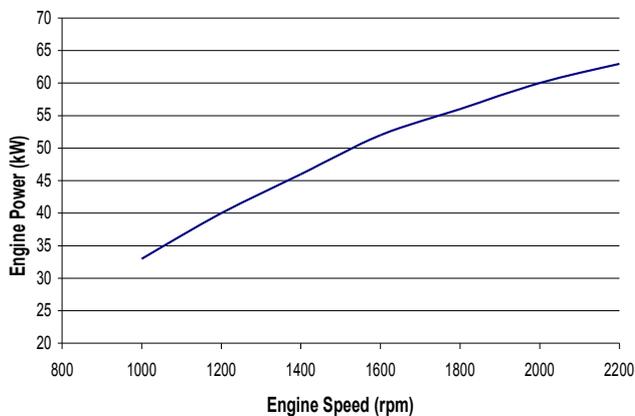
If we substitute all the values on the right hand side of equation (3), we get the same value for m as found using the plot above (*please check!!*).

Results from relationship (2) are presented in the following table:

Engine Speed (rpm)	800	1000	1200	1600	1800	2000	2200
Engine Power (kW)	22.9	28.6	34.4	45.8	51.6	57.3	63.0

The actual power curve for the 63 kW engine is shown in the graph below.

JCB Dieselmax 63kW Power Curve



The curve varies from the straight line plotted previously because efficiency, fuelling and torque demand vary with speed. Power and torque are related through the following formula:

$$\text{Power} = \text{Torque} \times \text{Angular Velocity} \dots (4)$$

We can approximate the power curve above by the following quadratic equation:

$$P = -0.0364 S^2 + 3.4429 S - 14.286 \dots (5)$$

where P is the power in kW and S is the engine speed in rpm.

The power of the JCB Dieselmax engine as calculated using equation (1) can be enhanced by increasing the air density, fuel:air ratio or efficiency for a given swept volume, engine speed and fuel.

A significant increase in engine power can be achieved by the use of *turbochargers*. A turbocharger utilises the energy from the exhaust gas flow to turn a compressor which compresses the ambient air at the engine intake and hence increases the density of the air entering the engine cylinders. Pressure ratios of 2 to 3 are common, and this can be translated, without any additional modifications, into an increase in engine power of 2-3 times.

EXTENSION ACTIVITY – 1:

Create a spreadsheet programme in Excel to plot automatically the power curve of an engine (equation 1) where the values for ρ_a , $\left[\frac{F}{A} \right]$, Q_{lhv} , η , etc. remain the same over the entire speed range as given on page 1. Calculate the engine power at each of the 14 speed points starting from 800 rpm to 2200 rpm at an interval of 200 rpm.

EXTENSION ACTIVITY – 2:

The engine power is now being increased through the use of turbocharger. In this turbocharged engine, the inlet pressure is 3 bar (1 bar is 100,000 Pa), the inlet temperature is 60°C and these are related to the density by the formula

$$\rho_a = \frac{\text{Pressure}}{R \times T}$$

Here, R is the universal gas constant with value 287J/kg.K and T is the temperature in degrees Kelvin. Calculate the new density and hence the new numerical values of engine power at 800, 1500 and 2200 rpm. Using the same spreadsheet program, explain the difference between the two curves you obtained from activities 1 and 2.

(Please remember to convert the temperature to Kelvin first and hence calculate ρ_a .)

EXTENSION ACTIVITY – 3:

The calculation for power includes a term for efficiency η . This efficiency is the product of three efficiencies within the engine. Find out more about these efficiencies and comment on their existence.

WHERE TO FIND MORE

1. *Basic Engineering Mathematics*, John Bird, 2007, published by Elsevier Ltd.
2. *Internal Combustion Engine Fundamentals*, John Heywood, 1988, published by McGraw Hill
3. *Introduction to Internal Combustion Engines (2nd edition)*, Richard Stone, 1992, published by Macmillan
4. <http://auto.howstuffworks.com/diesel.htm>



Alan Curtis – Research Engineer, JCB Power Systems Limited

Alan studied Mechanical Engineering at Loughborough University and graduated in 2006. Since then, Alan has worked for JCB Power Systems on the design and specification of engines to comply with future emissions legislation. Alan devotes a generous portion of his time to supporting younger engineers at various stages of their education.

INFORMATION FOR TEACHERS

The teachers should have some knowledge of

- Using empirical formulae for simple calculations
- How to find units using Dimensional Analysis
- How to find the equation of a straight line

TOPICS COVERED FROM “MATHEMATICS FOR ENGINEERING”

- Topic 1: Mathematical Models in Engineering
- Topic 4: Functions
- Topic 5: Geometry

LEARNING OUTCOMES

- LO 01: Understand the idea of mathematical modelling
- LO 04: Understand the mathematical structure of a range of functions and be familiar with their graphs
- LO 05: Know how 2-D and 3-D coordinate geometry is used to describe lines, planes and conic sections within engineering design and analysis
- LO 09: Construct rigorous mathematical arguments and proofs in engineering context
- LO 10: Comprehend translations of common realistic engineering contexts into mathematics

ASSESSMENT CRITERIA

- AC 1.1: State assumptions made in establishing a specific mathematical model
- AC 4.1: Identify and describe functions and their graphs
- AC 4.2: Analyse functions represented by polynomial equations
- AC 5.1: Use equations of straight lines, circles, conic sections, and planes
- AC 9.1: Use precise statements, logical deduction and inference
- AC 9.2: Manipulate mathematical expressions
- AC 9.3: Construct extended arguments to handle substantial problems
- AC 10.1: Read critically and comprehend longer mathematical arguments or examples of applications.

LINKS TO OTHER UNITS OF THE ADVANCED DIPLOMA IN ENGINEERING

- Unit-1: Investigating Engineering Business and the Environment
- Unit-4: Instrumentation and Control Engineering
- Unit-6: Investigating Modern Manufacturing Techniques used in Engineering
- Unit-7: Innovative Design and Enterprise
- Unit-8: Mathematical Techniques and Applications for Engineers
- Unit-9: Principles and Application of Engineering Science

ANSWERS TO EXTENSION ACTIVITIES

EA1: Please check the spreadsheet, data and graph online.

EA2: Density $\rho = 3.139 \text{ kg/m}^3$. Also check the spreadsheet, data and graph online.

EA3:

1. Mechanical efficiency, η_{mech} : accounting for losses caused by mechanical friction within the engine
2. Volumetric efficiency, η_{vol} : accounting for losses caused by restriction of the air inducted into engine by pipework, plenums, etc
3. Fuel conversion efficiency, η_{fuel} : accounting for losses caused by incomplete combustion of the fuel