

## INTRODUCTION

RWE npower is a leading integrated UK energy company and is part of the RWE Group, one of Europe's leading utilities. We own and operate a diverse portfolio of power plant, including gas-fired combined cycle gas turbine, oil, and coal fired power stations, along with Combined Heat and Power plants on industrial site that supply both electrical power and heat. RWE npower also has a strong in-house operations and engineering capability that supports our existing assets and develops new power plant. Our retail business, npower, is one of the UK's largest suppliers of electricity and gas.

In the UK RWE is also at the forefront of producing energy through renewable resources. npower renewables leads the UK wind power market and is a leader in hydroelectric generation. It developed the UK's first major off-shore wind farm, North Hoyle, off the North Wales coast, which began operation in 2003.

Through the RWE Power International brand, RWE npower sells specialist services that cover every aspect of owning and operating a power plant, from construction, commissioning, operations and maintenance to eventual decommissioning.

## SCENARIO

In thermal power plants, energy is extracted from steam under high pressure and at a high temperature. The steam is produced in a boiler or heat recovery steam generator and is routed to a steam turbine where it is partly expanded in a high pressure stage, extracting energy from the steam as it passes through turbine blades. The steam is then returned to the boiler for reheating to improve efficiency, after which it is returned to the turbine to continue expanding and extracting energy in a number of further stages. The steam turbine shaft rotates at 3000 revolutions per minute (rpm) (50 revolutions per second).



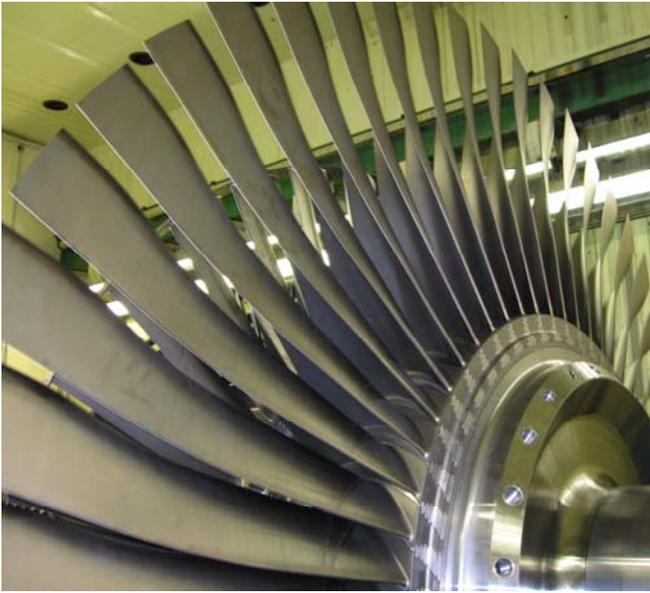
**Figure 1: Detailed view of turbine blades**

With blades (see Figure 1) rotating at such speeds, it is important that the fleet of steam turbines is managed to ensure safety and continued operation. If a blade were to fail in-service, this could result in safety risks and can cost £millions to repair and, whilst the machine is not generating electricity, it can cost £hundreds of thousands per day in lost revenue.

It is therefore important to understand the forces and resultant stresses acting on a blade and its root (where it is connected to the rotor disc) due to its rotational speed. Turbine blades are designed with margins of safety and the calculation of blade stresses gives an understanding of how they behave in-service under a variety of operating conditions. Knowledge of the forces and stresses is used when assessing the damage to a blade or assessing the effects of using different materials.

## PROBLEM STATEMENT

With the knowledge that an understanding of the forces and stresses acting on the turbine blades is of vital importance, in this exemplar we will calculate such a force acting on a last stage Low Pressure (LP) blade and root of a large steam turbine rotating at 3000 rpm in order to estimate the material stresses at the blade root. One such LP steam turbine rotor is shown in Figure 2 below:

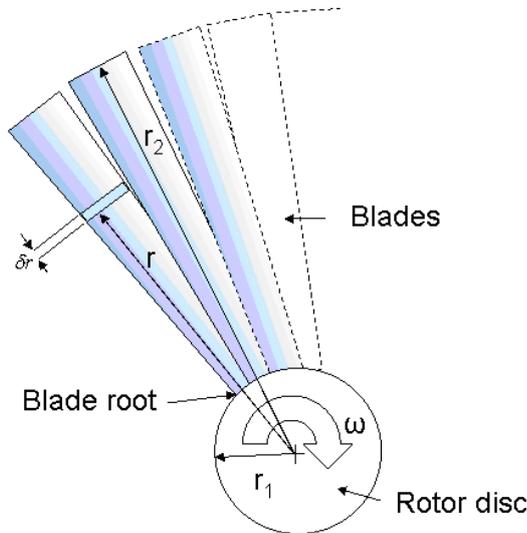


**Figure 2: Low Pressure steam turbine rotor**

### MATHEMATICAL MODEL

The **centripetal force** is the external force required to make a body follow a curved path. Any force (gravitational, electromagnetic, etc.) or combination of forces can act to provide a centripetal force. This force is directed inwards, towards the centre of curvature of the path.

A simplified 2D figure of the blades under discussion is shown in Figure 3.



**Figure 3: Simplified blade dimensions**

The general equation for centripetal force is:

$$F = m\omega^2 r \dots (1)$$

where  $m$  is the mass of the moving object,  $r$  is the distance of the object from the centre of rotation (the radius of curvature) and  $\omega$  is the angular velocity of the object.

In the case under consideration, we need to account for the fact that the mass of the blade is distributed over its length and the radius of curvature also changes along the length of the blade.

Consider a small segment of mass  $\delta m$ , of length having width  $\delta r$  at a distance  $r$  from the centre. Then the equation for the centripetal force  $\delta F$  on this small segment is given by:

$$\delta F = \delta m \omega^2 r \dots (2)$$

In practice, a blade tapers in thickness towards its tip; but, for simplicity, assuming the blade to have a constant cross sectional area  $A$  ( $\text{m}^2$ ) and material density  $\rho$  ( $\text{kg}/\text{m}^3$ ), we can write:

$$\delta m = \rho A \delta r$$

and equation (2) becomes:

$$\delta F = \rho A \omega^2 r \delta r$$

or formally:

$$dF = \rho A \omega^2 r dr \dots (3)$$

Let  $r_1$  be the radius of the rotor disc and  $r_2$  be the distance between the centre of the rotor disc and tip of the blade. Then, integrating equation (3) along the total length of the blade, the total centripetal force acting on the blade is given by:

$$F = \rho A \omega^2 \int_{r_1}^{r_2} r dr$$

So,

$$F = \rho A \omega^2 \left( \frac{r_2^2 - r_1^2}{2} \right) \dots (4)$$

We can convert the angular velocity from revolutions per minute (rpm) to radians per second using the following relationship:

$$\omega = \frac{\text{rpm} \times 2\pi}{60} \dots (5)$$

Knowing the values for the cross-sectional area, density, angular velocity and radii, we can then calculate the force on one blade. Once this force has been calculated, we can estimate the nominal stress  $\sigma$  on the blade root using the following relation:

$$\sigma = \frac{F}{A_{\text{root}}} \dots (6)$$

where  $A_{\text{root}}$  is the cross-sectional area of the blade root.

### CALCULATION WITH GIVEN DATA

We are given the following data:

- Angular velocity,  $\omega$  = 3000 rpm
- Blade cross-sectional area,  $A$  =  $0.0012 \text{ m}^2$
- Material density,  $\rho$  =  $7860 \text{ kg}/\text{m}^3$
- Blade tip radius,  $r_2$  = 1.6 m

Last stage (LP) blade length, i.e.  
 $(r_2 - r_1) = 0.9 \text{ m}$

Blade root cross-sectional area,  
 $A_{root} = 0.008 \text{ m}^2$

From equation (5), we can calculate the angular velocity in radians per second as follows:

$$\omega = \frac{3000 \times 2\pi}{60} = 100\pi = 314.2 \text{ rad/sec}$$

Also, from values given in the above table, we find that  $r_1 = r_2 - (r_2 - r_1) = 1.6 - 0.9 = 0.7 \text{ m}$ .

Substituting these values in equation (4), we get:

$$F = 7860 \times 0.0012 \times (314.2)^2 \times \left( \frac{(1.6)^2 - (0.7)^2}{2} \right)$$

This gives:

$$F = 963 \text{ kN/blade}$$

Using equation (6), we can now calculate the stress as follows:

$$\sigma = \frac{963 \times 10^3}{0.008} = 120 \times 10^6 \text{ N/m}^2 \text{ or Pa}$$

Equivalently,

$$\sigma = 120 \text{ N/mm}^2$$

### **CONCLUSION**

Steam Turbine rotors are designed with a significant factor of safety, the operating forces and stresses calculated above are normal for a last stage blade and under normal conditions this should not pose any risk to the blades integrity. Large steam turbine blades are manufactured from 12% chrome high alloy steels where maximum design stress values of 200-300 N/mm<sup>2</sup> are permissible.

This exemplar demonstrates that under normal operating conditions, stresses are well within acceptable limits. However, what the exemplar also shows are the large forces and stresses involved in such rotating machinery and how important factors such as design philosophy, manufacture and maintenance strategy are to ensure safe operation.

Last stage blades are exposed to adverse conditions within the steam turbine. The steam is expanded through the earlier stages and begins to condense and become wet as it reaches the last stage blades. Blades often have features that introduce higher stresses because of their geometry or shape. We mitigate these risks by inspecting last stage blades at a 4 year inspection

interval to look for signs of cracking and degradation.

In reality last stage blades have complex 3D designs of blade airfoils and blade roots and calculating stresses is done through advanced computer modelling techniques and simulation. Calculations like the one above provide engineers with information to start a discussion and a baseline to compare the computer analysis and simulation results to, these discussions and comparisons are just part of the daily job for mechanical engineers in the power industry.

### **DID YOU KNOW?**

- In a large 500 MW power boiler, steam is generated at a rate of approximately 420 kg/sec. Every hour this is a mass equivalent to over 100 large double-decker buses.
- The force on one blade as calculated in the example above is equivalent to the mass of 80 Mini Coopers on the end of one blade. There are typically 90 blades on the last rotor stage of a 500 MW LP steam turbine. The mass of one mini cooper is approximately 1200 Kg – quite astounding really!

### **EXTENSION ACTIVITY – 1:**

In the example above, what is the velocity of the blade tip in m/sec?

### **EXTENSION ACTIVITY – 2:**

The normal operation rotational speed is 3000 rpm. During an over-speed test, the turbine can run at 3300 rpm. How would this affect the forces and stresses?

### **EXTENSION ACTIVITY – 3:**

How do the blade forces and root stresses change with increasing blade length?

### **EXTENSION ACTIVITY – 4:**

Bearing in mind the assumption that the blade is of constant cross-sectional area along its length, how could the calculation above be simplified?

### **WHERE TO FIND MORE**

1. *Basic Engineering Mathematics*, John Bird, 2007, published by Elsevier Ltd.
2. *Engineering Mathematics*, Fifth Edition, John Bird, 2007, published by Elsevier Ltd.



**Gary – Steam Turbine Engineer, RWE npower Engineering**

He says:

*“I studied Aerospace Engineering before joining RWE npower as a graduate mechanical engineer on our Professional Engineers and Scientists Development Programme within the Turbine Generators Team. I am now a permanent member of the Steam Turbine Group looking after internal and external customers and providing technical solutions to a variety of mechanical issues on rotating plant applications.”*

## **INFORMATION FOR TEACHERS**

The teachers should have some knowledge of

- Centripetal forces and stress calculations
- Integration
- Unit conversion

## **TOPICS COVERED FROM “MATHEMATICS FOR ENGINEERING”**

- Topic 1: Mathematical Models in Engineering
- Topic 6: Differentiation and Integration

## **LEARNING OUTCOMES**

- LO 01: Understand the idea of mathematical modelling
- LO 06: Know how to use differentiation and integration in the context of engineering analysis and problem solving
- LO 09: Construct rigorous mathematical arguments and proofs in engineering context
- LO 10: Comprehend translations of common realistic engineering contexts into mathematics

## **ASSESSMENT CRITERIA**

- AC 1.1: State assumptions made in establishing a specific mathematical model
- AC 6.3: Find definite and indefinite integrals of functions
- AC 9.1: Use precise statements, logical deduction and inference
- AC 9.2: Manipulate mathematical expressions
- AC 9.3: Construct extended arguments to handle substantial problems
- AC 10.1: Read critically and comprehend longer mathematical arguments or examples of applications

## **LINKS TO OTHER UNITS OF THE ADVANCED DIPLOMA IN ENGINEERING**

- Unit-1: Investigating Engineering Business and the Environment
- Unit-3: Selection and Application of Engineering Materials
- Unit-4: Instrumentation and Control Engineering
- Unit-5: Maintaining Engineering Plant, Equipment and Systems
- Unit-6: Investigating Modern Manufacturing Techniques used in Engineering
- Unit-7: Innovative Design and Enterprise
- Unit-8: Mathematical Techniques and Applications for Engineers
- Unit-9: Principles and Application of Engineering Science

## **ANSWER TO EXTENSION ACTIVITIES**

**EA1:** Use the equation  $V = \omega r$  but remember  $V$  is in m/sec so some conversion of units may be required.

**EA2:** As the rotational speed increases to 3300rpm (10% overspeed) the forces and stresses increase by a factor of  $1.1^2$  ( $F, \sigma \propto \omega^2$ )

**EA3:** As the radius of the blade increases the blade forces and root stresses increase, for example a new design of a blade has a blade tip radius  $r_2$  of 2m, compare the stresses to the previous exercise to confirm the relationship between stresses and blade length. ( $F, \sigma \propto r$ )

**EA4:** Using a mean diameter  $r_{ave}$ , i.e. saying that  $F = m\omega^2 r_{ave}$ . In this formulation,  $m = \rho A(r_1 - r_2)$  and

$$r_{ave} = \frac{1}{2}(r_1 + r_2) \text{ so } F = \rho A(r_1 - r_2)\omega^2 \frac{1}{2}(r_1 + r_2) = \rho A\omega^2 \left( \frac{r_2^2 - r_1^2}{2} \right).$$