Steam Pipe Insulation
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Mechanical Engineering

INTRODUCTION

Steam pipes are very important in engineering application and are widely used. The main applications include household boilers, industrial steam generating plants, locomotives, steam engines, different building works, etc. to name but a few. Lack of proper insulation results in large energy losses which in turn cost a lot of money over time. Without proper insulation, the amount of energy lost can be 10 times greater than the energy being delivered through those pipes.

Insulation is defined as those materials or combinations of materials which retard the flow of heat energy by performing one or more of the following functions:

1. Conserve energy by reducing heat loss or gain

2. Control surface temperatures for personnel protection and comfort

3. Facilitate temperature control of a process

4. Prevent vapor flow and water condensation on cold surfaces

5. Increase operating efficiency of heating/ventilating/cooling, plumbing, steam, process and power systems found in commercial and industrial installations

6. Prevent or reduce damage to equipment from exposure to fire or corrosive atmospheres

The temperature range within which the term "thermal insulation" applies is from –73.3ºC (–100ºF) to 815.6ºC (1500ºF). All applications below –73.3ºC (–100ºF) are termed "cryogenic" and those above 815.6ºC (1500ºF) are termed "refractory".

In analogy to electrical resistance, the overall effect of an insulator can be described in terms of its "thermal resistance". The higher the thermal resistance the less the heat flow for a given temperature difference across the insulator, just as the higher the electrical resistance the less the current flow for a given potential difference across a resistor.

PROBLEM STATEMENT

An engineer wishes to insulate bare steam pipes in a boiler room to reduce unnecessary heat loss and to prevent people from burning themselves. After putting a thin layer of insulation material onto a pipe the engineer is surprised to find the heat loss actually increases! This is because there are two competing effects at work. The insulating material does, indeed, increase the thermal resistance to heat flow out of the pipe. However, it also increases the surface area that dissipates heat to the surrounding environment.

Initially, the increase in heat transfer area outweighs the increase in thermal resistance. As more insulation is added the heat loss reaches a maximum and then decreases as the thermal resistance eventually wins out. There is a critical radius of insulation at which the heat loss is a maximum. Calculate the value of this critical insulation radius and the radius beyond which the insulation starts to be effective as an insulator.

MATHEMATICAL MODEL
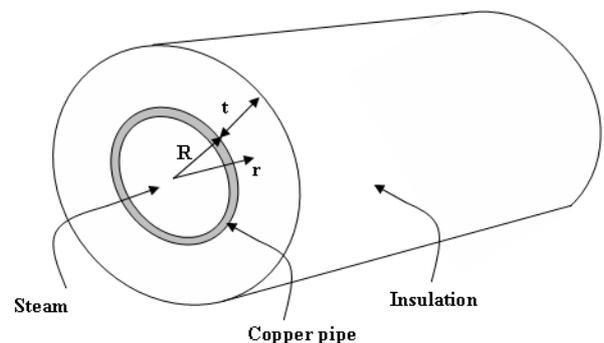
Figure-1: 3D Sketch of Copper Pipe with Insulation

Assume the steam is not superheated so that some steam will be condensing on the inside of the pipe. The entire inside of the pipe will be at a constant temperature corresponding to the saturation temperature of water, T_{sat} ('sat' representing 'saturation') at the steam pressure. The thermal conductivity of the copper pipe is many orders of magnitude larger than that of the insulation material, so we can assume the temperature drop through the thickness of the pipe is negligible, and that the temperature at the outside surface of the pipe (hence, the inside surface of the insulation) is also T_{sat}.

We'll assume the pipe is very long relative to its diameter, so heat flow is essentially one-dimensional, in the radial direction only. *Please note: one-dimensional heat transfer assumes that heat flows in a straight line, from the warm side of a component to the cold side, and perpendicular to the plane of the component.* Then, within the insulation, Fourier's law of heat conduction states that the heat flowing out over a length, L, of the pipe is given by:

$$Q = -kA_r \frac{dT_r}{dr} \quad \dots (1)$$

where Q is the rate of flow of heat [W], k is the thermal conductivity of the insulation [W/(m.degK)], T is temperature [K] and r is the radial distance [m]. The area is given by:

$$A_r = 2\pi \, r \, L \quad \text{… (2)}$$

If we define the heat loss per unit length of pipe by

$$P = \frac{Q}{L} \quad \text{… (3)}$$

Then by substituting (2) and (3) into (1) we get:

$$P = -2\pi \, k \, r \frac{dT_r}{dr} \quad \text{… (4)}$$

The heat transferred from the outside of the insulation to the surrounding air is given by Newton's law of cooling. This states that:

$$Q = hA_{R+t}(T_{R+t} - T_{air})$$

or

$$P = 2\pi \, h(R+t)(T_{R+t} - T_{air}) \quad \text{… (5)}$$

where T_{R+t} is the temperature at the outside surface of the insulation [K] and T_{air} is the temperature of the air [K]. The parameter, h, is called the *heat transfer coefficient* [W/(m².degK)].

By equating (4) and (5) and performing a little algebraic manipulation we get:

$$-\frac{dT_r}{dr} = \frac{h(R+t)(T_{R+t} - T_{air})}{k}\frac{1}{r} \quad \text{… (6)}$$

Separating variables and integrating, we find:

$$-\int_{T_{sat}}^{T_{R+t}} dT_r = \frac{h(R+t)(T_{R+t} - T_{air})}{k}\int_{R}^{R+t}\frac{dr}{r}$$

Performing integration on both sides, we get

$$T_{sat} - T_{R+t} = \frac{h(R+t)(T_{R+t} - T_{air})}{k}\ln\left(\frac{R+t}{R}\right)$$

Using equation (5), we can write

$$T_{sat} - T_{R+t} = \frac{P}{2\pi \, k}\ln\left(\frac{R+t}{R}\right) \quad \text{… (7)}$$

Rearranging (5) we can also write:

$$T_{R+t} - T_{air} = \frac{P}{2\pi \, h(R+t)} \quad \text{… (8)}$$

So, by adding (7) and (8) we get:

$$T_{sat} - T_{air} = \frac{P}{2\pi \, k}\ln\left(\frac{R+t}{R}\right) + \frac{P}{2\pi \, h(R+t)}$$

Rearranging for P, we get

$$P = \frac{2\pi (T_{sat} - T_{air})}{\left[\dfrac{1}{k}\ln\left(\dfrac{R+t}{R}\right) + \dfrac{1}{h(R+t)}\right]} \quad \text{… (9)}$$

Consider the example values to be as follows:

T_{sat} = 100°C = 373.15 deg K,

T_{air} = 20°C = 293.15 deg K,

R = 6 cm = 0.06 m,
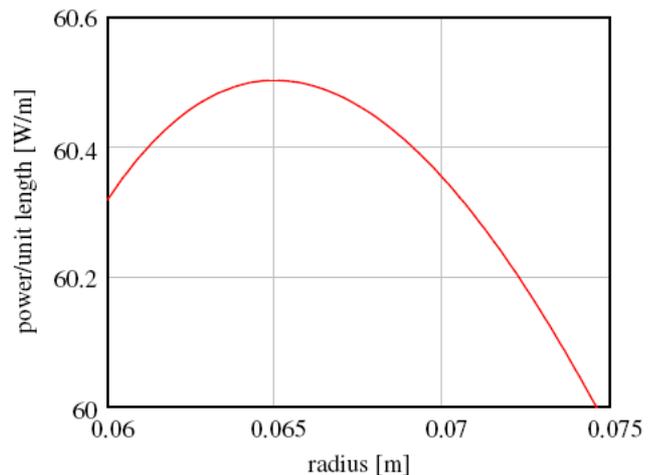
k = 0.13 W/(m.degK),

h = 2 W/(m².degK).

This gives the following function on substitution:

$$P = \frac{2\pi (373.15 - 293.15)}{\left[\dfrac{1}{0.13}\ln\left(\dfrac{0.06+t}{0.06}\right) + \dfrac{1}{2(0.06+t)}\right]} \quad \text{… (10)}$$

Plot a graph of P against radius $R+t$ to find the values of t at which P is a maximum and the value at which the insulation actually starts to insulate, in other words the value of t for which P is less than its value when $t = 0$.

Figure-2: Graph of Power per Unit Length vs. Radius

The graph clearly shows that P is a maximum with about 5 mm of insulation (for a total radius of ~65 mm), and that about 11 mm of insulation is needed (for a total radius of ~71 mm) before P is less than that for the bare pipe, i.e. for the insulation to actually insulate!

WHERE TO FIND MORE

1. *Basic Engineering Mathematics*, John Bird, 2007, published by Elsevier Ltd.

2. *Engineering Mathematics*, Fifth Edition, John Bird, 2007, published by Elsevier Ltd.

3. Details about Insulation Types and Material: http://micainsulation.org/standards/materials.htm

Dr. Alan Stevens – Specialist in Mathematical Modelling and Simulation (Retired, Rolls-Royce)

Spent 35 years as an industrial mathematician in the Submarines division of Rolls-Royce, dealing primarily with heat transfer and fluid-flow behaviour of the nuclear reactors used to power the Royal Navy's submarines.

INFORMATION FOR TEACHERS

The teachers should have some knowledge of

- Fourier's Law of Heat Conduction
- Integration using the Method of Separation of Variables
- How to plot the graphs of simple functions using Excel® or Autograph or Mathcad

TOPICS COVERED FROM "MATHEMATICS FOR ENGINEERING"

- Topic 1: Mathematical Models in Engineering
- Topic 4: Functions
- Topic 6: Differentiation and Integration

LEARNING OUTCOMES

- LO 01: Understand the idea of mathematical modelling
- LO 04: Understand the mathematical structure of a range of functions and be familiar with their graphs
- LO 06: Know how to use differentiation and integration in the context of engineering analysis and problem solving
- LO 09: Construct rigorous mathematical arguments and proofs in engineering context
- LO 10: Comprehend translations of common realistic engineering contexts into mathematics

ASSESSMENT CRITERIA

- AC 1.1: State assumptions made in establishing a specific mathematical model
- AC 1.2: Describe and use the modelling cycle
- AC 4.1: Identify and describe functions and their graphs
- AC 4.2: Analyse functions represented by polynomial equations
- AC 6.1: Calculate the rate of change of a function
- AC 6.3: Find definite and indefinite integrals of functions
- AC 6.4: Use integration to find areas and volumes
- AC 9.1: Use precise statements, logical deduction and inference
- AC 9.2: Manipulate mathematical expressions
- AC 9.3: Construct extended arguments to handle substantial problems
- AC 10.1: Read critically and comprehend longer mathematical arguments or examples of applications.

LINKS TO OTHER UNITS OF THE ADVANCED DIPLOMA IN ENGINEERING

- Unit-1: Investigating Engineering Business and the Environment
- Unit-3: Selection and Application of Engineering Materials
- Unit-4: Instrumentation and Control Engineering
- Unit-5: Maintaining Engineering Plant, Equipment and Systems
- Unit-6: Investigating Modern Manufacturing Techniques used in Engineering
- Unit-7: Innovative Design and Enterprise
- Unit-8: Mathematical Techniques and Applications for Engineers
- Unit-9: Principles and Application of Engineering Science