AC characteristics and AC waveform addition

Electrical and electronic engineering

- Voltmeter
- Sinusoidal and non-sinusoidal waveforms
- Amplitude
- Time period
- Frequency
- Instantaneous values
- Peak-to-peak value
- Combining waveforms
- Root mean square (rms) value
- Phase angle
- Form factor
- Average value
- Peak value

Mathematics

- Square and square roots
- Basic arithmetic

Topic area and summary of methods used

- To study a diagrammatic representation of several AC waveforms.
- To understand the definitions of principle characteristics: amplitude, period time, frequency, instantaneous, peak/peak-to-peak, root mean square (rms), average values and form factor, and their relevance.
- To determine principle characteristics for a sine wave.
- To apply the addition of sinusoidal voltages and extension to active noise cancellation.

Prerequisites

None

Problem statement

AC waveforms are common in engineering and are found in a wide range of applications such as AC power delivery, instrument measurements, control signals and audio applications. Periodic AC waveforms can be characterised by a number of key parameters that allow an engineer to produce effective designs that work with them.

Additionally, AC waveforms can be manipulated and added to produce new waveforms, a process that can be used to produce active noise cancellation that can protect the ears from noisy plants or prevent external sounds from detracting from spoken instructions or music sent to headphones.

How can AC signals be characterised and added and how can these be useful?

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Activity 1 – Discussion

Figure 1 shows four different AC waveforms.

1. What characteristics do the four waveforms have in common? What characteristics are different?
2. Suggest names to describe the waveforms.
3. Which waveform represents how the voltage from an AC mains outlet varies?

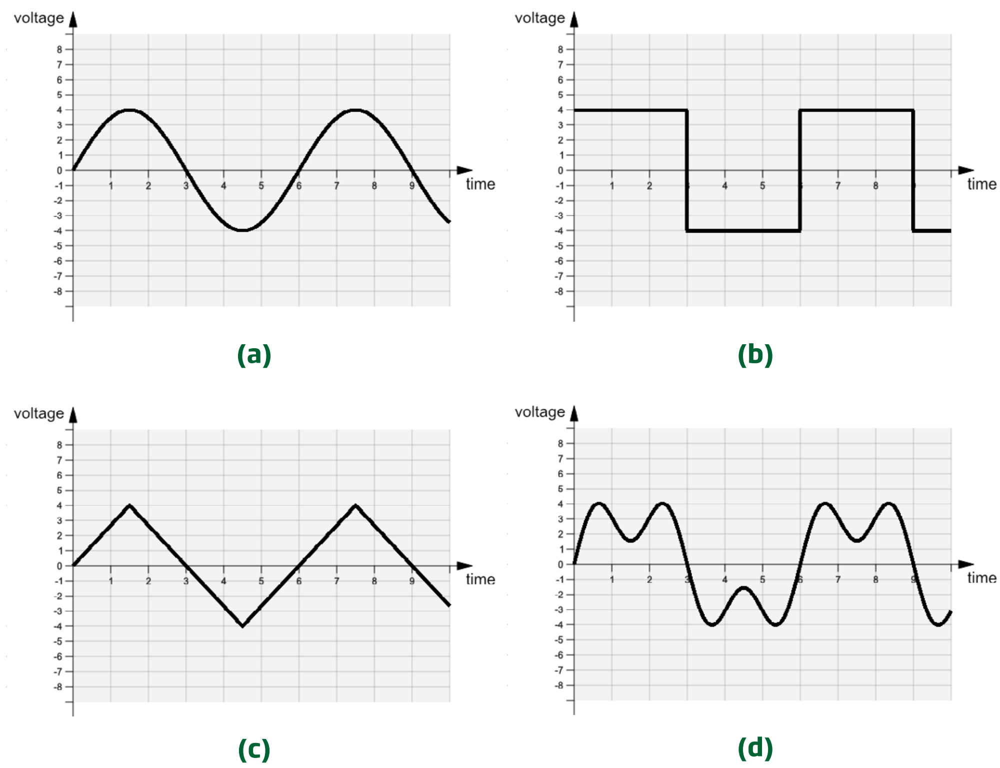


Figure 1a-d
Example AC waveforms

Background – instantaneous, peak, peak-to-peak values, time period and frequency.

The diagrams in Figures 1a, b, c and d can be used to show some characteristic values.

- The instantaneous value is the voltage measured at a particular point in time. For the four waveforms shown, this will typically be different for each of them.
- The peak value is the largest value the waveform can have. In **Figure 1**, all the waveforms have a peak value of 4 V.
- The peak-to-peak value is the difference in values from the most positive to the most negative. In **Figure 1**, all four waveforms have a peak-to-peak value of 8 V (+4 V is the most positive, −4 V is the most negative and the difference between these is 8 V).
- The time period of a waveform is the time over which the waveform repeats. In **Figure 1** all four waveforms have a time period of 6 seconds.
- The frequency of a waveform is the number of complete waveforms that are completed in one second. For a waveform with a time period T in seconds, the frequency, f, is given in hertz (Hz) by:

$$f = \frac{1}{T}$$

Activity 2 – Characteristic values

A simple DC source connected to a voltmeter gives a constant value of the measured voltage, which can be used to characterise the DC source; for example, 9 V for a 9 V cell.

The waveforms are connected to the voltmeters shown in **Figures 2(a) and 2(b)**. The meter in **Figure 2(a)** can show positive and negative voltages with respect to a reference voltage. The meter in **Figure 2(b)** has a built-in rectifier and so only shows absolute voltages with respect to a reference voltage.

For each of the two meters, describe what they would show for:

1. The waveform varying slowly enough for the voltmeter to follow the voltage changes.
2. The AC waveform varying too fast for the voltmeter to follow changes and so the meter displays the average value.

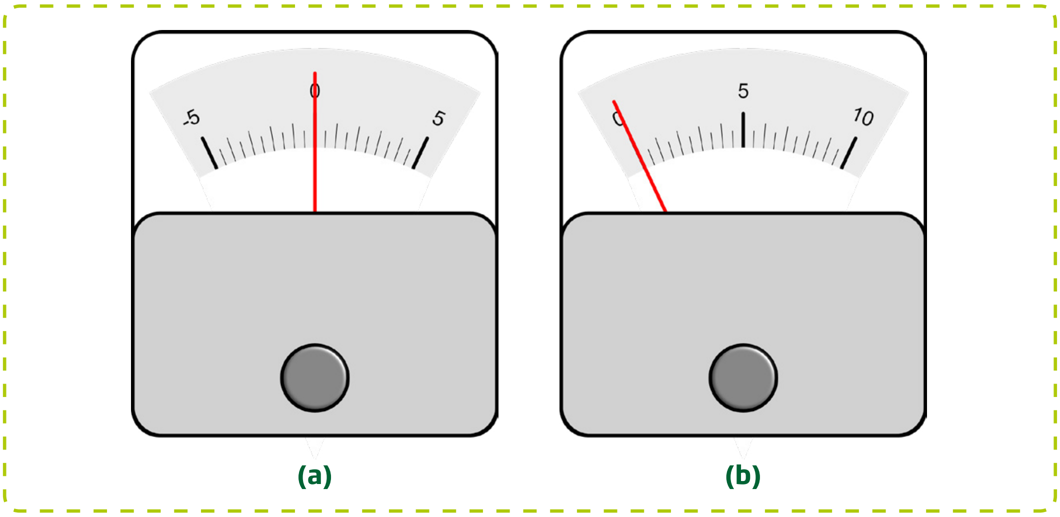


Figure 2

Interactive

The resource ACmeters can be used to simulate the possible meter and waveform combinations discussed in activity 2.

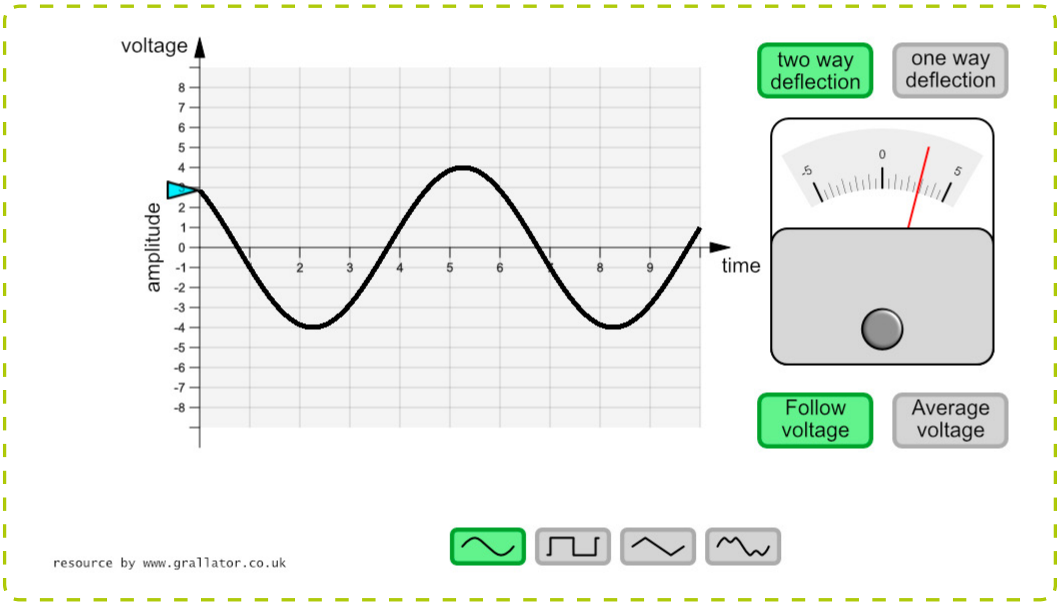


Figure 3
Screen shot of resource

Background – the root mean square (RMS) value

When a voltage V passes through a component of resistance R, the power dissipated is given by:

$$P = \frac{V^2}{R}$$

When the voltage is continuously varying, as in an AC supply, it might seem plausible that the average power dissipated is given by the square of the average voltage divided by the resistance, where the average voltage V_{ave} could be measured, for example, as in Activity 2:

$$P_{ave} = \frac{V_{ave}^2}{R}$$ **(Note: this is wrong!)**

However, this is wrong and does not give the correct average power dissipated – do not use the above!

To see why, consider the sinusoidal voltage shown in **Figure 4** passing through a component with resistance $10\ \Omega$.

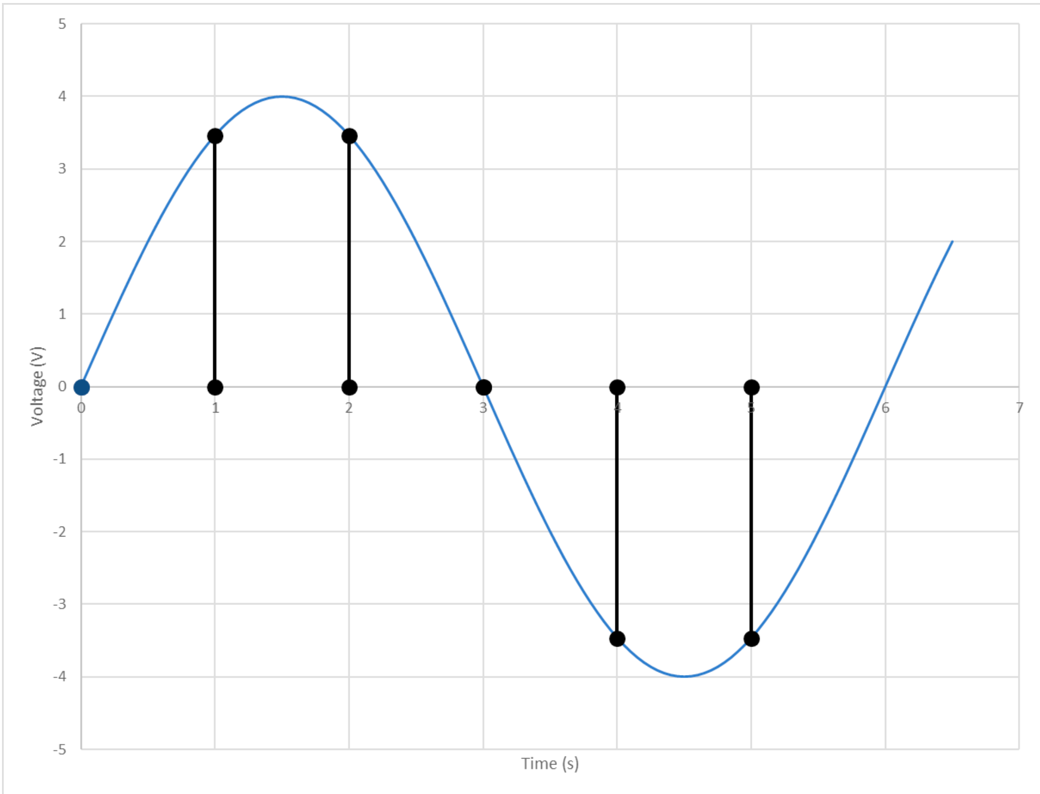


Figure 4
Averaging points on a sinusoidal wave

Table 1 shows the instantaneous and calculated average voltage over one cycle for two-way deflection and one-way deflection. It also shows the instantaneous power and average power over one cycle.

Time (s)	Instantaneous voltage (V)	Instantaneous absolute voltage (one-way deflection)	Instantaneous power $P = \frac{V^2}{R}$
0	0.0000	0.0000	0.0000
1	3.4641	3.4641	1.2000
2	3.4641	3.4641	1.2000
3	0.0000	0.0000	0.0000
4	−3.4641	3.4641	1.2000
5	−3.4641	3.4641	1.2000
Average $\left(= \frac{sum}{6}\right)$	**Average of instantaneous voltage** V_{avg} **= 0.0000**	**Average of instantaneous absolute voltage** V_{avg} **= 2.3094**	**Average of instantaneous power** P_{avg} **= 0.8000**

Table 1 Instantaneous and average values of the wave in Figure 4

The second column gives an average value of the instantaneous voltage of $V_{avg} = 0.0$. Using this value to calculate the average power dissipated gives $P_{ave} = \frac{V_{ave}^2}{R} = \frac{0^2}{10} = 0$. This predicts that the component dissipates no power, which is obviously wrong as it has a non-zero resistance.

The third column gives an average value of the instantaneous absolute voltage of $V_{avg} = 2.3094$. Using this value to calculate the average power dissipated gives $P_{ave} = \frac{V_{ave}^2}{R} = \frac{2.3094^2}{10} = 0.5333$ W. It was stated previously that this is wrong, which will be explained soon.

The fourth column finds an average of the actual powers dissipated, and gives a value of $P_{ave} = 0.8$ W. Notice, this is higher than the value of 0.5333 W calculated by averaging the voltage and then using it to calculate power. As the calculation of $P_{ave} = 0.8$ W is based on averaging power calculations at different times, it is expected that this value should be more representative of the true power dissipated. Notice that it is significantly higher than the previous value of 0.5333 W.

The result of $P_{ave} = 0.8$ can be worked backwards to find an equivalent voltage to give this power:

$$P_{ave} = \frac{V_{equiv}^2}{R}$$
$$V_{equiv} = \sqrt{P_{ave} \times R}$$
$$V_{equiv} = \sqrt{0.8 \times 10}$$
$$V_{equiv} = \sqrt{8}$$
$$V_{equiv} = 2.8284$$

This voltage value is called the root mean square (RMS) voltage. ***It is the DC equivalent voltage that gives an equivalent power dissipation.***

The average of the instantaneous absolute voltage using N points (N = 6 in the table above) is calculated using:

$$V_{ave} = \frac{V_1 + V_2 + \cdots V_N}{N}$$

However, it can be shown that the RMS voltage is calculated by finding the square root of the average of the square voltages:

$$V_{RMS} = \sqrt{V_{ave}^2} = \sqrt{\frac{V_1^2 + V_2^2 + \cdots + V_N^2}{N}}$$

For the example in the table:

$$V_{RMS} = \sqrt{\frac{0 + 3.4641^2 \; + 3.4641^2 \; 0 + 3.4641^2 \; + 3.4641^2}{6}}$$

$$V_{RMS} = \sqrt{\frac{48}{6}} = \sqrt{8} = 2.8284$$

as found above in the calculation of V_{equiv}.

The RMS voltage gives the power dissipated as:

$$P_{ave} = \frac{V_{RMS}^2}{R}$$

The ratio of the RMS to the simple average is called the form factor:

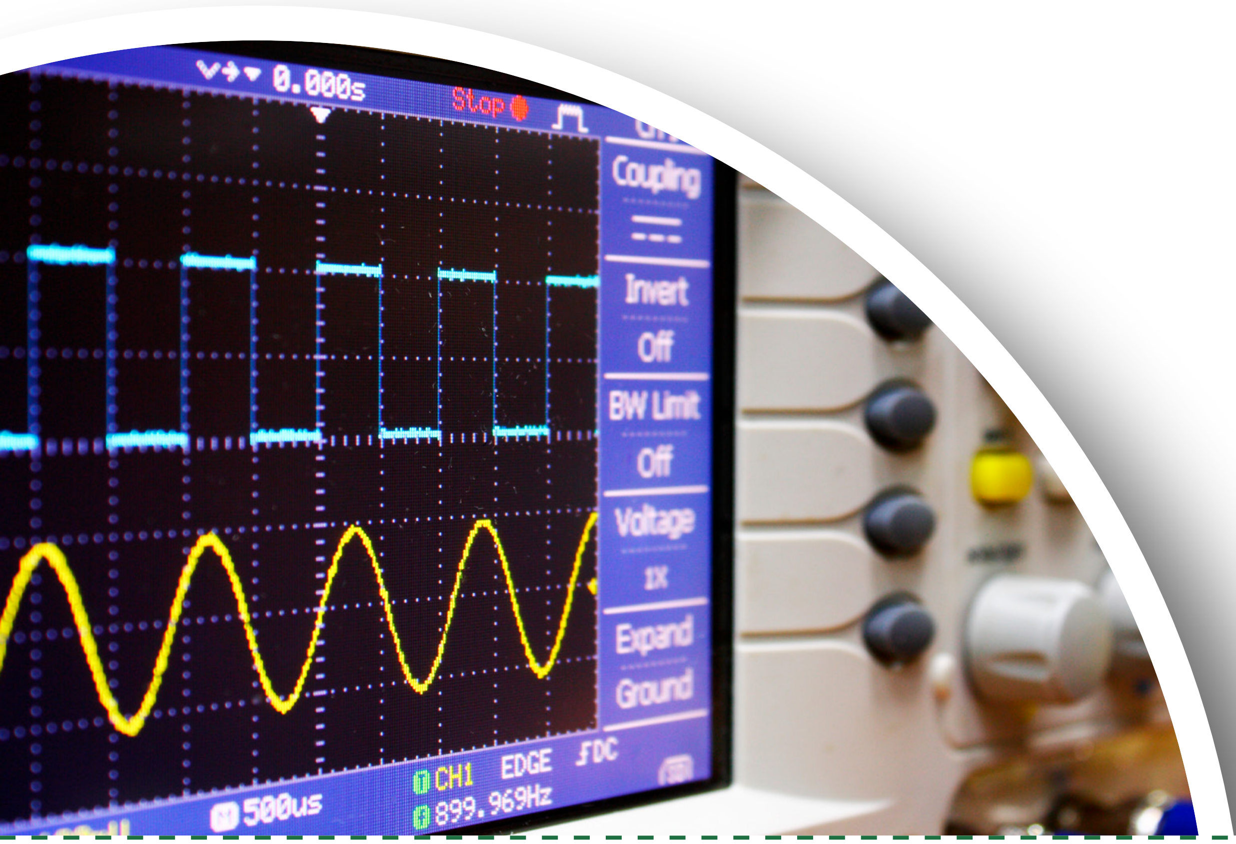
$$form\ factor = \frac{RMS}{average}.$$

The form factor is required when an analogue meter that responds to the rectified average of a measured quantity is being used. By multiplying the metered average reading by the form factor, you can obtain the RMS value and can assess the power dissipated.

In the example, the peak voltage is 4 V. The ratio of RMS voltage to peak voltage is:

$$\frac{2.8284}{4} = 0.7071$$

This ratio is for a sine wave – other waveforms will, in general, have different values.



Activity 3 – Interactive

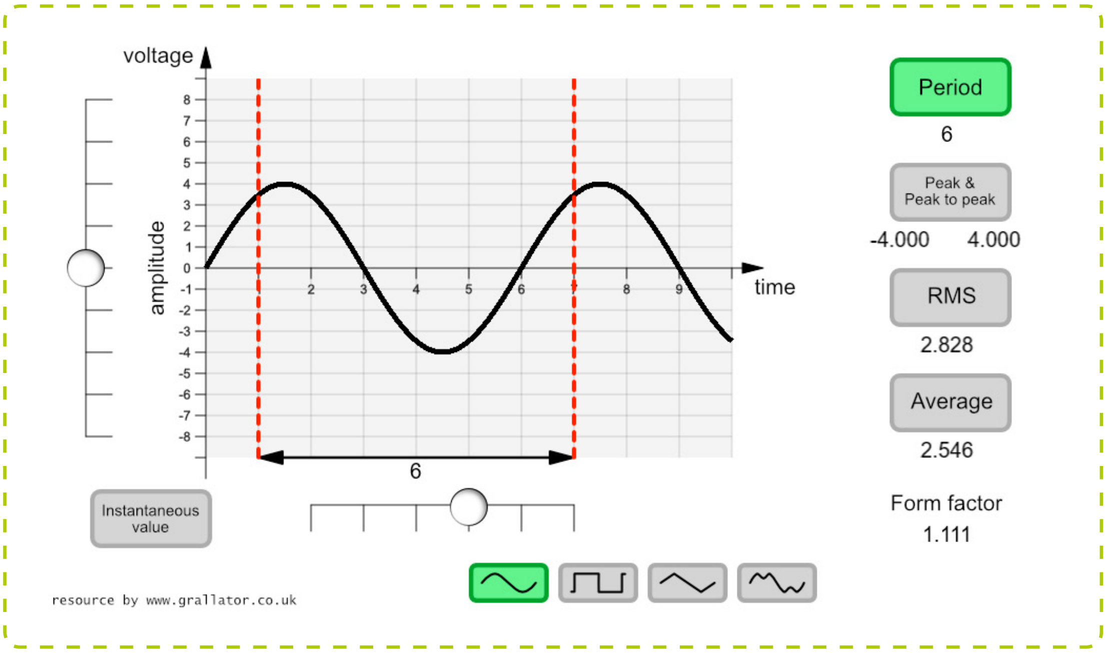


Figure 5
Screen shot of resource

The resource **ACwaveforms** can be used to investigate the properties of the example waveforms and compare how they change when the waveform changes, or when waveform properties, such as period and amplitude, change.

1 For each of the waveforms, use the left-hand slider to set the amplitude to 4 V and fill in the first three blank columns in the table (rms, average and form factor). Do a calculation in the final column to verify the value of the form factor.

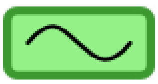
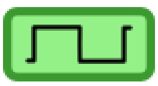
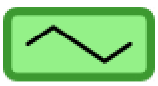
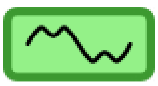
Waveform	RMS	Average	Form factor	Calculated value of $\frac{RMS}{average}$
				
				
				
				

Table 1
Comparison of AC waveform characteristics

2 For each of the waveforms, how does the ratio of RMS to peak, average to peak and form factor change as the amplitude and period change?

Activity 4 – Interactive

Two (or more) waveforms can be combined by simultaneously sending their signals to a common output. When this occurs, the instantaneous values of the two waveforms sum to give a resultant output.

When the two waveforms have positive or negative values, these combine to produce a larger resultant signal. When one waveform has a positive value and the other has a negative value, these combine to produce a smaller resultant signal. Also, electronics can be used to delay on waveform with respect to another so that the two signals are not *in phase*. This alters how two waveforms interact.

The resource **ACaddition** can be used to investigate the addition of two identically-shaped waveforms of different amplitudes and with different phase differences (the blue and the red traces on the screen). The resultant waveform is shown in black. Experiment with the amplitude, phase and waveform to see how the two waves are added to give a resultant. Select the "Trace lines" option and click-drag the mouse on the plot area to show the two components and resultant at a particular instant in time.

1 What must be satisfied for the two waveforms being added to produce a zero signal?

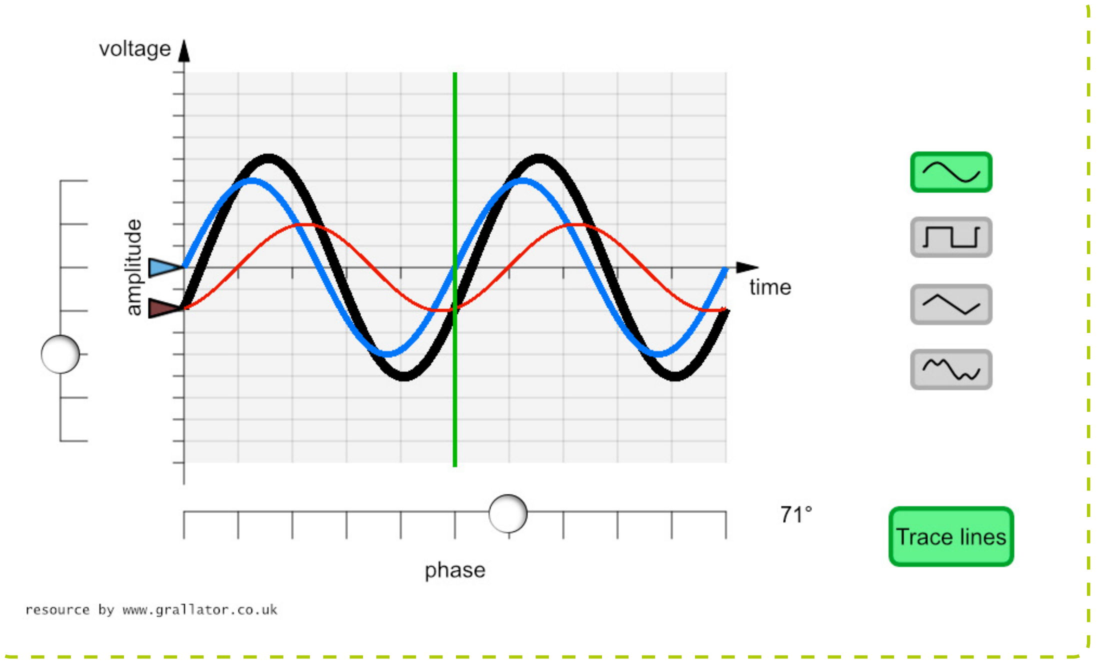
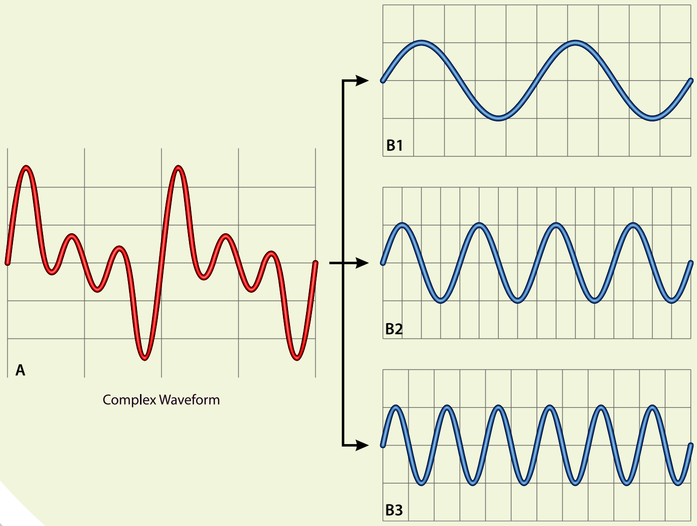


Figure 6
Screen shot of resource

In the case where a complex waveform represents an undesirable external sound, electronics can be used to process the signal and feed it back through the speakers in a set of earphones so that the two signals cancel out in your ear (in this case, whether the signal is positive or negative indicates whether the sound wave is causing your ear drum to move in or out).

This is the principle of noise cancelling headphones. If an additional signal is added, such as speech or music, this will be unaffected as the positive external noise will be immediately subtracted, leaving only the speech or music, and no distracting external noise.

Note, in the above, the addition could be made using phasors, as in the resource "AC phasors".



Stretch and challenge activity

The average and RMS voltage of a sinusoidal waveform can be calculated by subdividing the shape as shown in **Figure 7**, and calculating the following:

$$V_{ave} = \frac{|V_1| + |V_2| + \cdots + |V_N|}{N}$$

$$V_{RMS} = \sqrt{V_{ave}^2} = \sqrt{\frac{V_1^2 + V_2^2 + \cdots + V_N^2}{N}}$$

Where $|x|$ means the absolute (positive) value of x. Use a spreadsheet to show that, as the number of subdivisions used increases, the ratio of the average to the peak tends to $\frac{2}{\pi} = 0.637...$, and the ratio of the RMS to the peak tends to $\frac{1}{\sqrt{2}} = 0.707...$

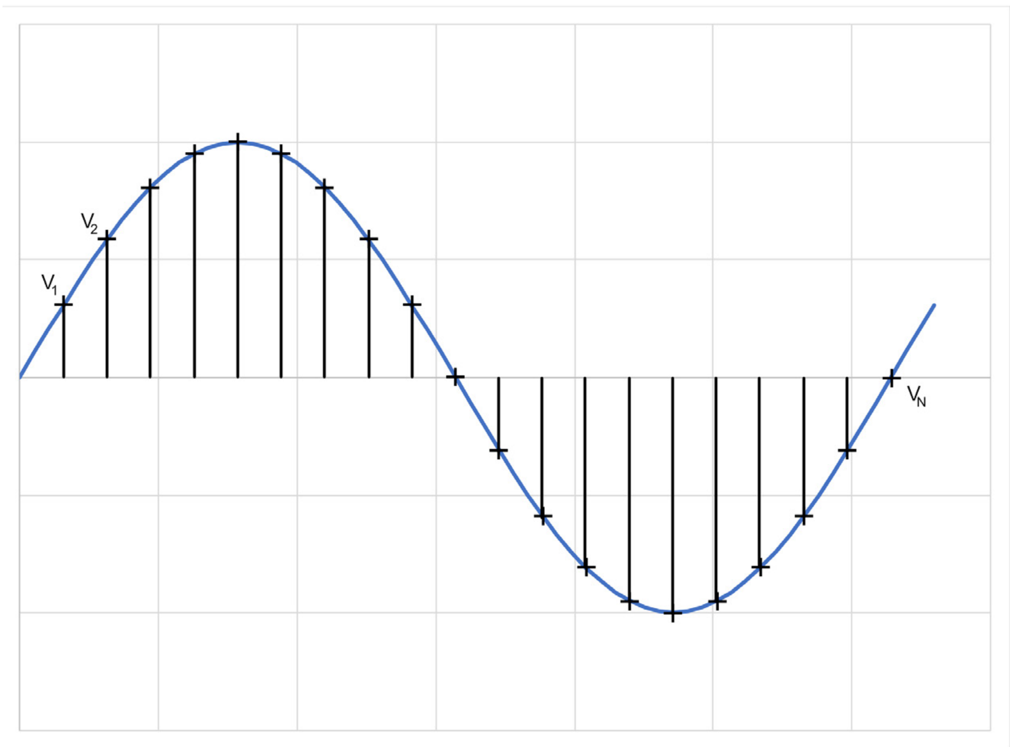


Figure 7 Finding the average and RMS values of a sinusoidal waveform

Notes and solutions

Activity 1

1. *Example common characteristics*
   The voltage alternates in polarity from positive to negative and back again.
   The maximum voltage is 4 V.
   The minimum voltage is 4 V.
   The time for which the voltage is positive is the same time for which it is negative.
   The pattern repeats after every 6 units of time. This is called the *period* of the waveform.

   Different characteristics
   The way in which the voltage varies with time is different for each waveform.
2. (a) Sine wave
   (b) Square wave
   (c) Triangular wave
   (d) Complex wave (as it's not a simple geometric shape).
3. The voltage from the AC mains varies as the sine wave in **Figure 1(a)**.

Activity 2

1. The voltmeter in **Figure 2(a)** would deflect to the right on the positive portions of the waveform and to the left on the negative portions. In this case, the meter is showing the *instantaneous value* of the AC waveform. The maximum deflection to the left to the maximum deflection to the right gives the waveform's *peak-to-peak* range.

   The voltmeter in **Figure 2(b)** can deflect only to the right but this does not mean that the signal would give values only for the positive half of the waveform as there is a built-in rectifier. This means that the meter shows absolute values (so negative values are shown as positive). The maximum deflection shows the waveform's *peak* value.
2. The voltmeter in **Figure 2(a)** will average over positive and negative values. As the waveforms are positive and negative for equal values and times over a single period (see answer to activity 1, question 1), the average is zero, so the meter would read zero.

   The voltmeter in **Figure 2(b)** measures only absolute values so would show an average value over a single period for a waveform where all the negative values are made positive, which is non-zero. For example, waveform **1(d)** would look like the solid line in **Figure 8**.

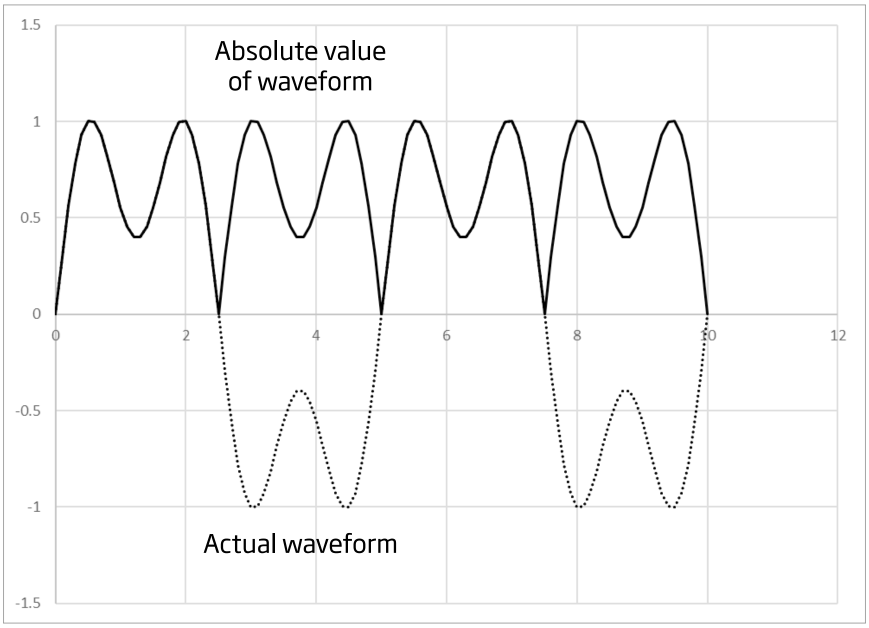


Figure 8
Absolute value of a waveform

Activity 3

1.

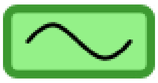
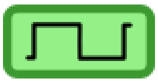
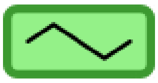
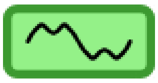
Waveform	RMS	Average	Form factor	Calculated value of $\frac{RMS}{average}$
	2.828	2.546	1.111	1.111 (3 d.p.)
	4.000	4.000	1.000	1.000
	2.309	2.000	1.155	1.155 (3 d.p.)
	2.893	2.696	1.073	1.073 (3 d.p.)

Table 3 Comparison of AC waveform characteristics – results

2. The RMS, average and form factor do not change when the period changes. The values of RMS and average do change when the amplitude changes, however, the form factor is constant. The ratio of the RMS to peak and average to peak do not change.

Activity 4

1. For the two waveforms to combine to produce a zero signal, the values at any point in time must be equal and opposite in sign. In terms of the resource, the two waves must have the same amplitude and have a phase difference of +180° or –180°.

Stretch and challenge activity

A spreadsheet with the formula shown in **Figure 9** will give output as shown in **Figure 10**. Note, only the first two rows of the spreadsheet are required – the rest are produced using copy and paste. For N = 20, the average value is not quite at the desired limit of 0.637, however, the RMS value is. Increasing the number of points considered will improve the answer. Note, for 20 points the step is PI()/10, for 40 points the step will be PI()/20.

	x	sin(x)	abs(sin(x))	sin^2(x)
1	=PI()/10	=SIN(E8)	=ABS(F8)	=F8^2
2	=E8+PI()/10	=SIN(E9)	=ABS(F9)	=F9^2
3	=E9+PI()/10	=SIN(E10)	=ABS(F10)	=F10^2
4	=E10+PI()/10	=SIN(E11)	=ABS(F11)	=F11^2
5	=E11+PI()/10	=SIN(E12)	=ABS(F12)	=F12^2
6	=E12+PI()/10	=SIN(E13)	=ABS(F13)	=F13^2
7	=E13+PI()/10	=SIN(E14)	=ABS(F14)	=F14^2
8	=E14+PI()/10	=SIN(E15)	=ABS(F15)	=F15^2
9	=E15+PI()/10	=SIN(E16)	=ABS(F16)	=F16^2
10	=E16+PI()/10	=SIN(E17)	=ABS(F17)	=F17^2
11	=E17+PI()/10	=SIN(E18)	=ABS(F18)	=F18^2
12	=E18+PI()/10	=SIN(E19)	=ABS(F19)	=F19^2
13	=E19+PI()/10	=SIN(E20)	=ABS(F20)	=F20^2
14	=E20+PI()/10	=SIN(E21)	=ABS(F21)	=F21^2
15	=E21+PI()/10	=SIN(E22)	=ABS(F22)	=F22^2
16	=E22+PI()/10	=SIN(E23)	=ABS(F23)	=F23^2
17	=E23+PI()/10	=SIN(E24)	=ABS(F24)	=F24^2
18	=E24+PI()/10	=SIN(E25)	=ABS(F25)	=F25^2
19	=E25+PI()/10	=SIN(E26)	=ABS(F26)	=F26^2
20	=E26+PI()/10	=SIN(E27)	=ABS(F27)	=F27^2
		sum/N	**=SUM(G8:G27)/20**	
		sqrt(sum/N)		**=SQRT(SUM(H8:H27)**

Figure 9
Excel formula

	x	sin(x)	abs(sin(x)	sin^2(x)
1	0.314159	0.309016994	0.309017	0.095492
2	0.628319	0.587785252	0.587785	0.345492
3	0.942478	0.809016994	0.809017	0.654508
4	1.256637	0.951056516	0.951057	0.904508
5	1.570796	1	1	1
6	1.884956	0.951056516	0.951057	0.904508
7	2.199115	0.809016994	0.809017	0.654508
8	2.513274	0.587785252	0.587785	0.345492
9	2.827433	0.309016994	0.309017	0.095492
10	3.141593	1.22515E-16	1.23E-16	1.5E-32
11	3.455752	-0.309016994	0.309017	0.095492
12	3.769911	-0.587785252	0.587785	0.345492
13	4.08407	-0.809016994	0.809017	0.654508
14	4.39823	-0.951056516	0.951057	0.904508
15	4.712389	-1	1	1
16	5.026548	-0.951056516	0.951057	0.904508
17	5.340708	-0.809016994	0.809017	0.654508
18	5.654867	-0.587785252	0.587785	0.345492
19	5.969026	-0.309016994	0.309017	0.095492
20	6.283185	-2.4503E-16	2.45E-16	6E-32
		sum/N	**0.631375**	
		sqrt(sum/N)		**0.707107**

Figure 10
Excel values

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