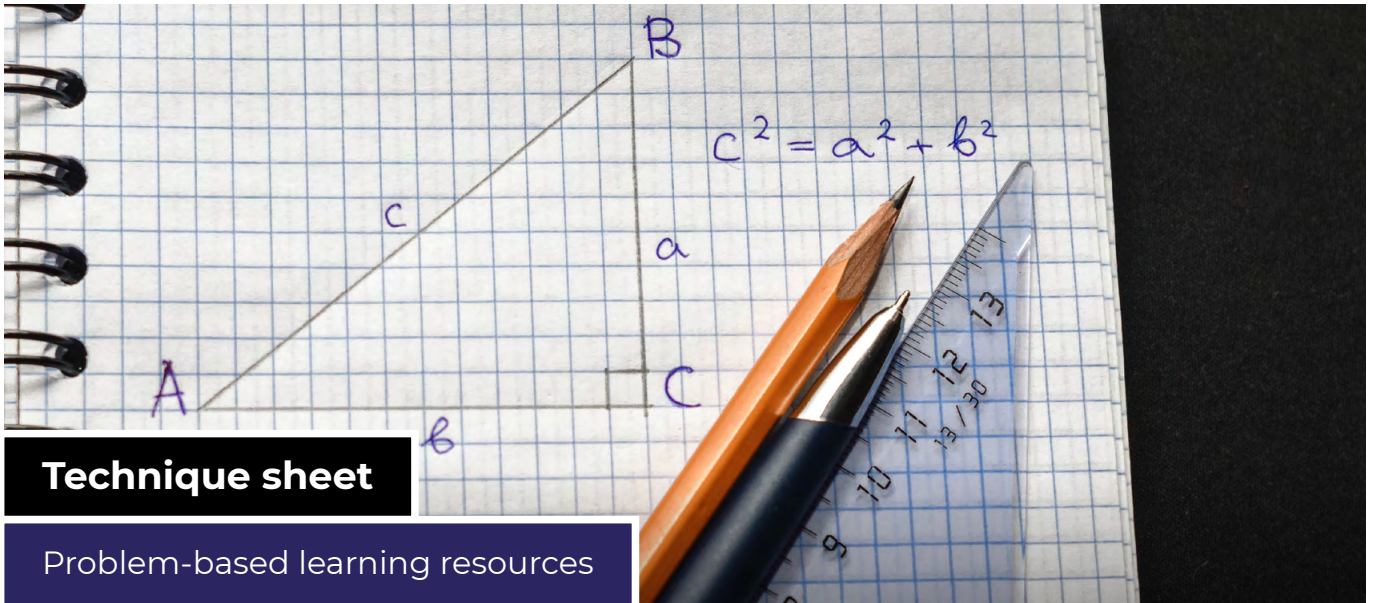


4

Working with triangles



Technique sheet

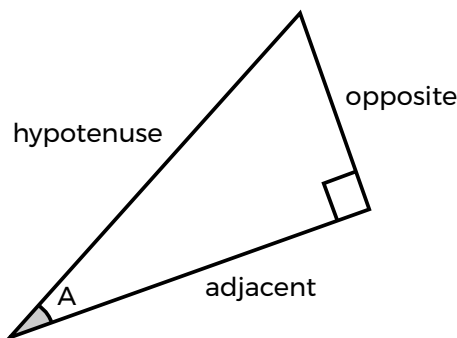
Problem-based learning resources

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How do I know which side is which in a triangle?

Imagine a triangle with a right angle in one corner and one of the other angles labelled A. The longest side is called the hypotenuse (hyp). The side opposite the angle A is called the opposite (opp) and the side next to it is called the adjacent (adj). When angle A gets larger the opposite side gets longer. Mathematicians can predict the ratio of different side lengths for every angle in a triangle.

$$\sin(A) = \frac{\text{opp}}{\text{hyp}} \quad \cos(A) = \frac{\text{adj}}{\text{hyp}} \quad \tan(A) = \frac{\text{opp}}{\text{adj}}$$



You can find the values of sin (pronounced sine), cos and tan on a scientific calculator or phone app. For example, to find the sin of 40° we would write sin(40). On a calculator you type 40 then press the

sin button. In an app press the sin button then type 40. It gives a long decimal value, so write it to a suitable level of accuracy e.g. 2 decimal places. Check that you get sin(40) = 0.75 on your device.

- Make sure you find the right angle and use it to find the hypotenuse. Then identify the angle you need to calculate. Now you can see which side is which and which trigonometric ratio might be useful.

How do I find the length of a side?

A cliff railway rises at 35° up a 30 m high cliff. This gives us the angle (35°) and the length of the opposite side (30 m) in a giant imaginary triangle. But how long is the track which forms the hypotenuse? Pick the trigonometric ratio that contains your known (opp) and the unknown (hyp) – this is $\sin(A) = \frac{\text{opp}}{\text{hyp}}$.

Put the values into the formula to make an equation $\sin(35) = \frac{30}{x}$. Then rearrange the formula to

$$\text{give } x = \frac{30}{\sin(35)}$$

In your calculator divide 30 by sin(35) to get the answer



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$x = 52.30$ m to 2 decimal places. That tells you how much track you need to build the railway.

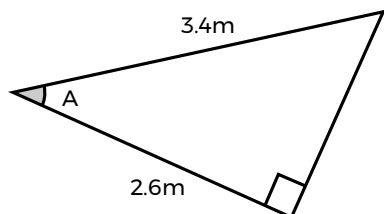
- Make sure you identify the sides and angle involved, choose the correct formula and put in the values. Rearrange the formula and do the calculation. Remember that it is a length, so remember to put the units.

How do I find the value of an angle?

Again, choose your formula depending on what you know and want to find out. In this right angled triangle you know the length of the hypotenuse (3.4 m) and the adjacent side (2.6m). The ratio which includes hyp and adj is $\cos(A) = \frac{\text{adj}}{\text{hyp}}$.

Put the values into the formula: $\cos(A) = \frac{2.6}{3.4}$ and calculate the answer.

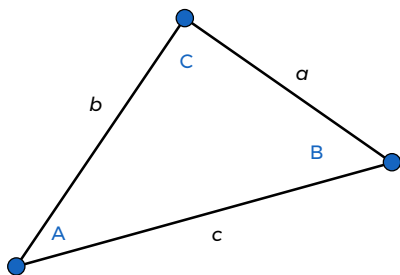
This gives us $\cos(A)$ as 0.77647 ... Work this out on your calculator/app and then you can find the angle which has this value of \cos using the \cos^{-1} button or INV COS. You should get the answer 40.12° to 2 decimal places.



- Make sure you choose the ratio you need to find your unknown and remember it is an angle so the units are degrees.

How can I deal with triangles without a right angle?

Look at what you already know and choose the formula to find the value you need. The cosine rule works where you know one angle and the two sides next to that angle. The sine rule works when you know two angles and the sides opposite them. These are not right-angled triangles so the sides are labelled differently. The side length a is opposite the angle A , length b is opposite the angle B and so on

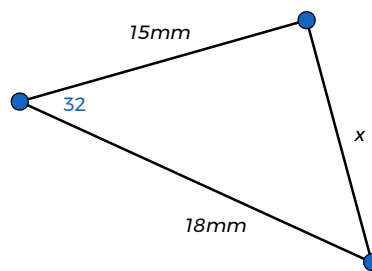


Cosine rule: $a^2 = b^2 + c^2 - 2bccos(A)$

Sine rule: $\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$

For example, if you know an angle and the side lengths next to it and need to find the side opposite the angle use the Cosine rule. The angle of 32° is opposite the side of x , so, in the formula these

should be A and a . The other two sides b and c are 15 mm and 18 mm, it doesn't matter which is which. So, put the values into $a^2 = b^2 + c^2 - 2bccos(A)$



$$x^2 = 15^2 + 18^2 - 2 \times 15 \times 18 \times \cos(32)$$

$$x^2 = 225 + 324 - 540 \times 0.848 \dots$$

$$x^2 = 98.519 \dots$$

$$x = 9.93 \text{ mm}$$

- Decide whether to use the sine or cosine rule according to which information you have. Pair up sides and opposite angles as A and a etc. Put the values into the formula. Then use your calculator, keeping all values without rounding until the final step when you can round e.g. to 2 decimal places and put in the units.



Check yourself

You should be able to answer these questions easily after reading this sheet.

- What is a right angle?
- What is the sin of an angle in a triangle?
- What rule do you use to find side lengths in a triangle without right angles?



Taking it further

These activities will deepen your understanding of this topic.

- In large scale projects, like map making, navigation or building, measuring the angle needs a special tool called a clinometer. You can make a decent one and then work out the height of a tall building by walking a measured distance away and measuring the angle to the top.
- A garden shed has a roof with a slope of 15° so that water runs off when it rains. The shed is 2 m wide and 3 m long. Allowing for a 10 cm overlap on both sides how long does a piece of water membrane need to be to cover the width of the roof? The membrane is sold in 10 m rolls that are 75 cm wide. How many sheds could you cover from 1 roll?