Centre of gravity of composite bodies

Topic areas

Mechanical engineering:

- Centre of gravity
- Forces
- Moments

Mathematics:

- Ratios
- Algebra

Prerequisites

None

Problem statement

The centre of gravity of an object is the average position of the weight of an object.

The motion of an object can be completely described by the translation of the centre of gravity, and by rotations about the centre of gravity.

When an object is static, its tendency to tip, or resist tipping, is determined by the position of the centre of gravity relative to the base upon which the object is resting.

How can you determine the position of the centre of gravity of an object?

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Background

The mass of an object is a measure of the amount of matter it contains, and has the SI unit of the kilogram (kg). An object that has mass feels a gravitational force which attracts it to other objects that have mass. This force is called the object's weight and is measured in Newtons (N). As the scale of the earth is large compared with many everyday objects, the weight force acting on an object, F, can be related to its mass, m, through the approximating expression

$$F = mg$$

where g is the acceleration due to gravity, usually taken to be 9.81 ms^{-2} at the earth's surface.

As stated in the problem statement, the centre of gravity of an object is the average position of the weight of an object, i.e. the average position of all the weight forces acting on a body if that body were considered as a large collection of small mass elements connected together. The position of the centre of gravity therefore depends on the shape and composition of an object.

When a force acts on a body the body will accelerate in the direction of the force unless there is a balancing force to oppose it. Similarly, if a force acts on an object that is in contact with a pivot, it can make it turn or tip. This is called a moment. As with linear acceleration, a moment will cause an angular acceleration about a pivot unless there is a balancing moment.

Activity 1 – Discussion

The position of the centre of gravity depends on the shape and composition of an object. In many engineered designs ballast weight can also be added to shift the centre of gravity to a more desirable position. Look at the image in Figure 1 of an aeroplane. Discuss where you might ideally want the centre of gravity to be.

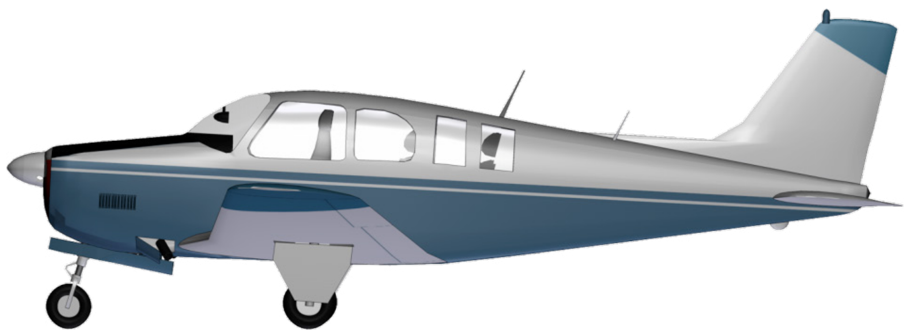


Figure 1

Activity 2

This activity will only consider laminae, i.e. those whose thickness is negligible compared with its other dimensions. The centre of gravity of simple regular objects can usually be determined by symmetry; for example, where are the centres of gravity of the two objects shown in Figure 2?

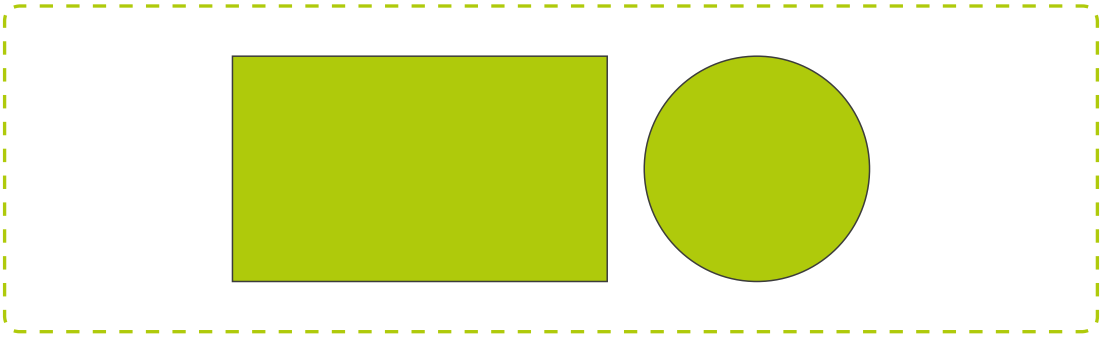


Figure 2

For more complex shapes, symmetry may give an indication, but not the exact location; for example, discuss what symmetry tells you about the location of the centres of gravity of the objects shown in Figure 3?

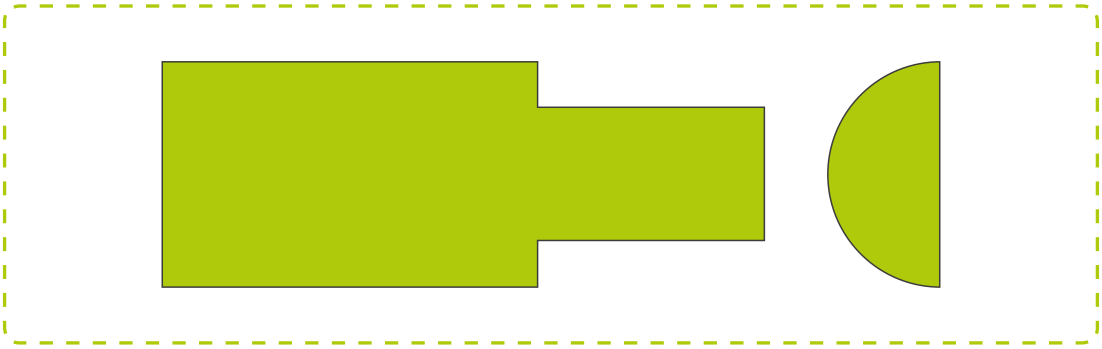


Figure 3

Complex shapes

Many complex shapes can be easily broken down into simpler shapes which have a known centre of gravity. These are called composite shapes. As the centre of gravity gives the position through which the force of gravity acts, such composite figures can be considered as being made up of point masses at these locations and the centre of gravity of the composite figure is given by the centre of gravity of the point masses.

Activity 3 – Shape decomposition

Break down the shape shown in Figure 4 into two simpler shapes with known centres of gravity.

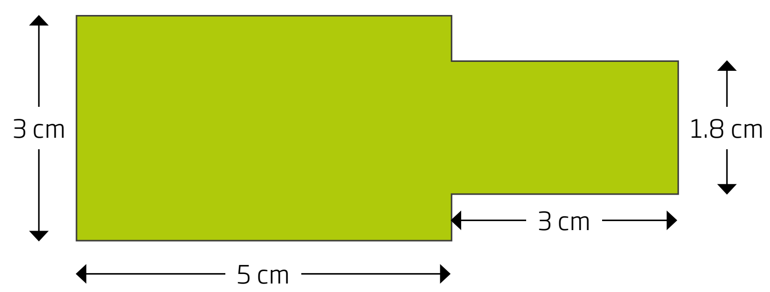


Figure 4

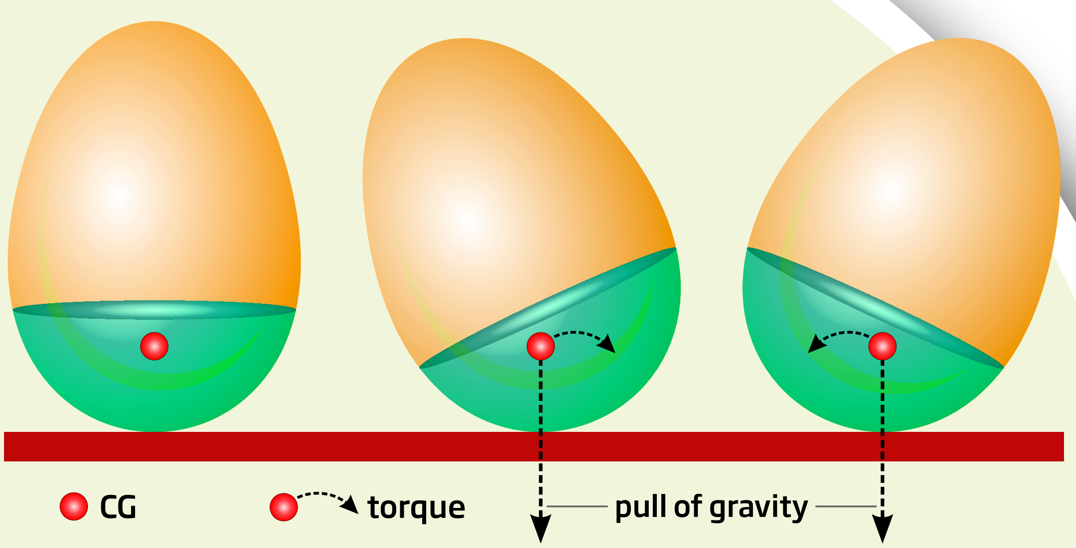
You could make a model of this out of thick paper or cardboard. If you do, you will be able to calculate the mass of each of the two simple shapes if you know how much a given area of the paper or cardboard weighs. Luckily the mass per unit area of paper is usually given on its packaging; for example paper may be described as 80gsm. This stands for 80 grammes per square metre, i.e. a sheet of this paper that is 1 m by 1 m will have a mass of 80 g.

1. What is the distance between the two centres of gravity?
2. Find the areas and masses of the two simpler shapes assuming a model is made using 80 gsm paper.

   Hint: remember the shape has dimensions given in centimetres whereas the mass is given for a square metre.
3. Calculate the ratio of

   a. The areas of the two simpler shapes
   b. The masses of the two simpler shapes

   What do you notice about the results?



Activity 4 – Balancing masses on an arm

The resource **balance-1** shows two masses on the end of a light arm (this acts like a class 1 lever), see Figure 5.

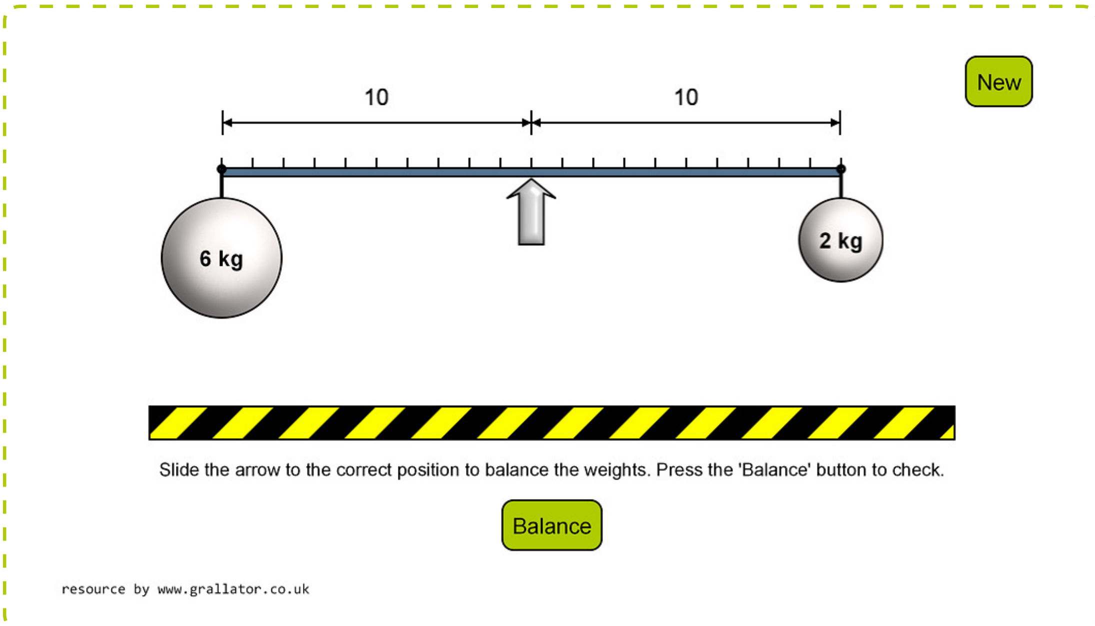


Figure 5
Screen shot of resource

1. Experiment with the simulation and find the balance point of the two masses on the arm using trial and error. This point represents the centre of gravity of the two point masses.
2. What do you notice about the ratio of the masses and the ratio of the distances they are from the position of the centre of gravity when the beam is balanced (as indicated by the position of the fulcrum or pivot point in the resource)?

Background - moments

The moment of a force is given by the size of the force multiplied by the distance it acts from a pivot point, $M = fd$. It is measured in Nm.

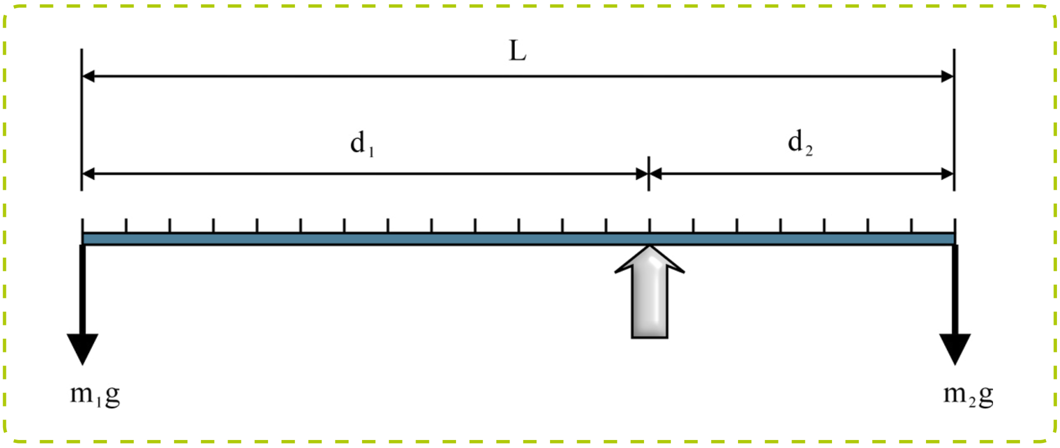


Figure 6
Forces and moments on a balanced beam.

In Figure 6, there are two moments acting on the beam:

- The moment produced by the force on the left, M_1, is given by the force acting, m_1g, multiplied by the distance from the pivot point, d_1, i.e. $M_1 = m_1gd_1$. This moment wants to turn the beam in an anticlockwise sense; it is an anticlockwise moment.

- The moment produced by the force on the right, M_2, is given by the force acting, m_2g, multiplied by the distance from the pivot point, d_2, i.e. $M_2 = m_2gd_2$. This moment wants to turn the beam in a clockwise sense; it is a clockwise moment.

To prevent the beam from turning the moments must be in balance, i.e. the total clockwise moment about the pivot point must be balanced by an equal total anti-clockwise moment.

Anti-clockwise moment: $M_1 = m_1gd_1$

Clockwise moment: $M_2 = m_2gd_2$

Balanced when: $m_1gd_1 = m_2gd_2 \Rightarrow m_1d_1 = m_2d_2$

Or, rearranged $m_1d_1 = m_2d_2 \Rightarrow \frac{m_1}{m_2} = \frac{d_2}{d_1}$

i.e. the ratio of the masses is equal to the inverse ratio of their distances from the pivot point, as discovered experimentally in question 2 of Activity 4.

Stretch and challenge activity

The shape in Activity 2, Figure 4, is made from a lamina which has a mass of 2 grammes per square centimetre of area (2 grammes.cm^{-2}).

In Activity 3 you broke this shape down into two simpler shapes and calculated their individual areas and masses based on the mass per unit area.

Note, this activity uses a different mass per unit area. The centre of gravities of the two shapes can be considered as masses at the end of a light arm that connects them.

In Activity 4 you found the balance point of two masses and discovered that the balance occurs when the moments of the weights about the pivot point are equal.

This balance point gives the position of the centre of gravity of the composite shape, see Figure 7.

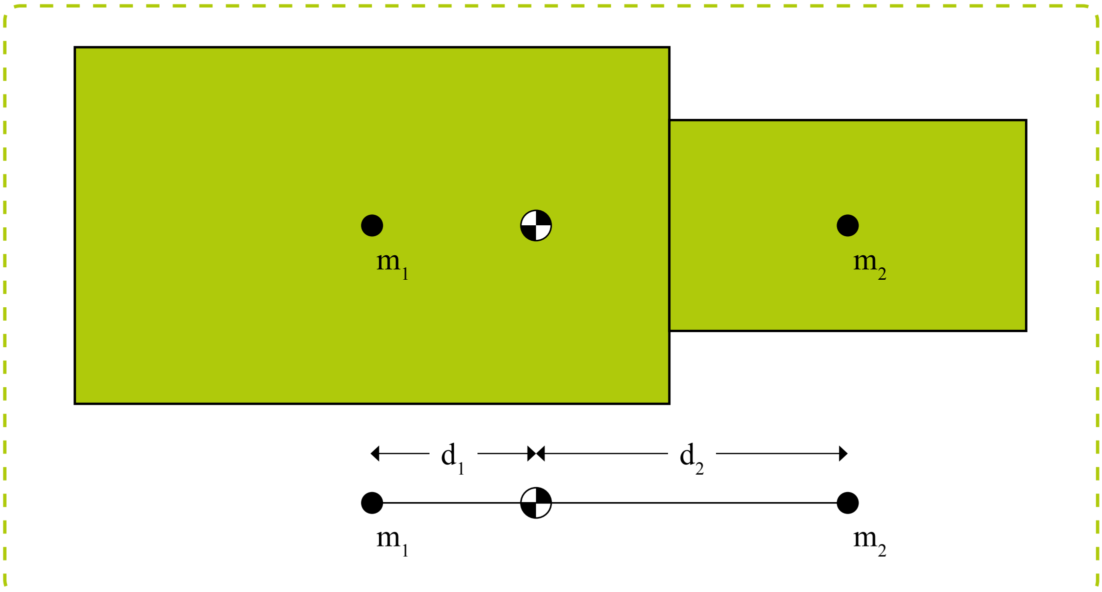


Figure 7

Use the results from Activities 3 and 4 to calculate the position of the centre of gravity of the shape.

Notes and solutions

Activity 1

There are two aspects to the position of the centre of gravity for the aeroplane; stability on the ground and stability in the air. On the ground, the centre of gravity must be between the front and rear landing wheels. If it were not the plane would tip over.

As the front wheel is very far forward it is unlikely the plane will tip forward. However, the rear landing wheels are about one-third the way along the aeroplane. Having a centre of gravity behind this point will tip the plane nose up onto its tail.

In the air, if the centre of gravity is too far forward the plane will tend to nose dive towards the ground requiring the pilot to fly the plane with a 'nose-up' attitude. This increases air-drag. If the centre of gravity is too far back the plane will have a tendency to nose-up, slowing down the plane and possibly putting the wings in a position where they stop generating lift, a condition known as a stall.

The ideal position is just forward of the centre point at which the wings generate lift (the centre of lift) as this gives the best stability and controllability in flight.

Note, flight stability is a complex topic – only basic principles are stated here!

Activity 2

The lines of symmetry give the locations of the centres of gravity at the centres of the simple objects.

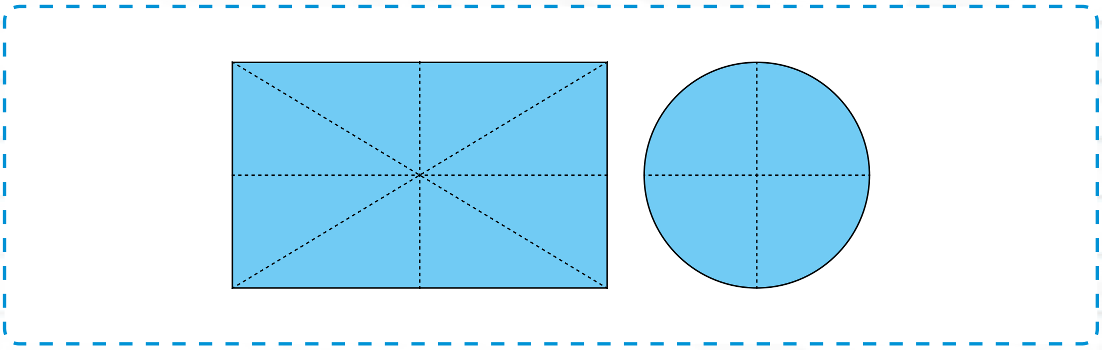


Figure 2 with some lines of symmetry shown.

The more complex shapes have one line of symmetry and the centre of gravity must lie on this. However, it is not clear where along this line the location actually is.

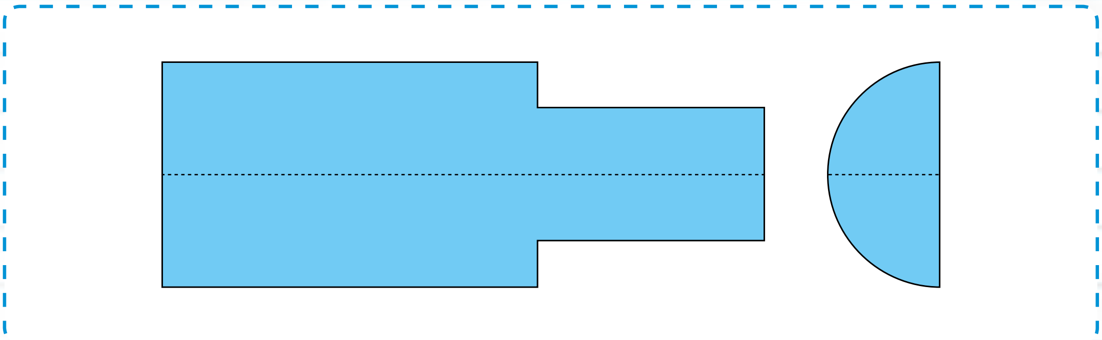


Figure 3 with lines of symmetry.

Activity 3 - Shape decomposition

The shape in Figure 4 breaks down into two rectangles as shown in Figure 8.

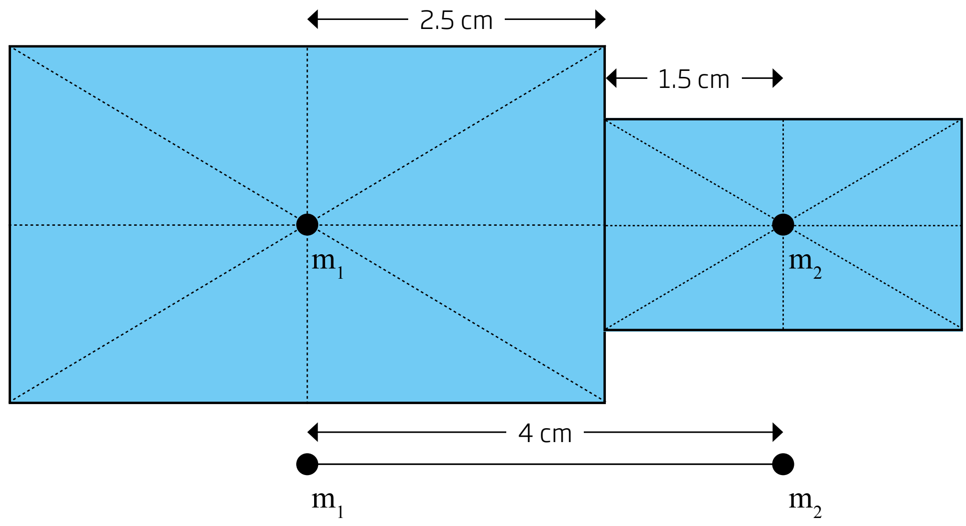


Figure 8
Breakdown of Figure 4.

1. The distance between the two centres of gravity is 4 cm.

2. The mass of each rectangle is given by its area multiplied by a mass per unit area of material. If you make a model rectangle out of thick paper or cardboard, the mass per unit area is usually given on its packaging; for example, paper may be described as 80gsm. This stands for 80 grammes per square metre, i.e. a sheet of this paper that is 1 m by 1 m will have a mass of 80 g.

   Area of larger rectangle: $A_1 = 5 \times 3 = 15\text{cm}^2$

   Area of smaller rectangle: $A_2 = 3 \times 1.8 = 5.4\text{cm}^2$

   Making a model out of 80 gsm paper (with a mass per unit area of 80 gm^{-2} = 8×10^{-3} g.cm^{-2}) gives the following masses:

   Mass of larger rectangle: $m_1 = A_1 \times 8 \times 10^{-3} = 15 \times 8 \times 10^{-3} = 0.12$ grammes

   Mass of smaller rectangle: $m_2 = A_2 \times 8 \times 10^{-3} = 5.4 \times 8 \times 10^{-3} = 0.0432$ grammes

3. Ratio of areas: $\frac{A_1}{A_2} = \frac{15}{5.4} = \frac{2.78}{1}$ (2d.p.) (or 25 : 9)

   Ratio of masses: $\frac{m_1}{m_2} = \frac{0.12}{0.0432} = \frac{2.78}{1}$ (2d.p.) (or 25 : 9)

   The ratio of masses is equal to the ratio of areas (as expected as the mass is given by a constant multiplier applied to the area).

Activity 4 - Balancing

1. Trial and error should lead to the conclusion that the balance point is always closer to the heavier weight. Further, it has to be much closer when there is a larger difference between the two weights.
2. When the beam is balanced you will notice that the ratio of the masses equals the inverse ratio of their distances from the centre of gravity (pivot point).

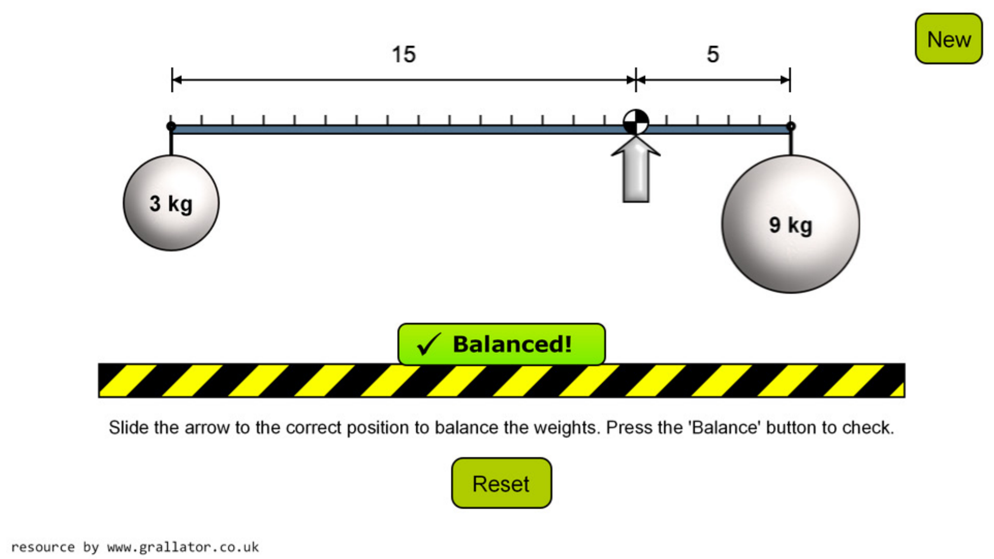


Figure 9

For example, in Figure 9, the ratio of the mass on the left to the mass on the right is $\frac{3}{9} = \frac{1}{3}$.

The ratio of the distance of the mass on the left from the centre of gravity to the distance of the mass on the right from the centre of gravity is $\frac{15}{5} = \frac{3}{1}$.

Some of you may have arrived at a balanced position by noticing that the product of the left mass value and its distance from the balance point must be equal to the product of the right mass value and its distance from the balance point. This solving the problem using moments, which is described by the next section of the main text.

Stretch and challenge activity

Mass of larger rectangle: $m_1 = 15\text{cm}^2 \times 2 \text{ grammes.cm}^{-2} = 30 \text{ grammes}$

Mass of smaller rectangle: $m_2 = 5.4\text{cm}^2 \times 2 \text{ grammes.cm}^{-2} = 10.8 \text{ grammes}$

The distance between the individual centres of gravity is 4 cm (see Figure 8) should be divided in the inverse ratio of the masses to find the balance point, $\frac{10.8}{30}$. This gives two equations

$$\frac{d_1}{d_2} = \frac{10.8}{30} = 0.36 \qquad (1)$$

$$d_1 + d_2 = 4 \qquad (2)$$

From (1): $d_1 = 0.36 d_2$

Substituting this into (2):

$$0.36 d_2 + d_2 = 4$$
$$1.36 d_2 = 4$$
$$d_2 = 2.94 \quad (2 \text{ d.p})$$

Using this value back in (2):

$$d_1 + d_2 = 4$$
$$d_1 + 2.94 = 4$$
$$d_1 = 4 - 2.94$$
$$d_1 = 1.06 \quad (2 \text{ d.p})$$

Solving gives $d_1 = 1.06$, $d_2 = 2.94$ (to 2 d.p.). Using this gives the centre of gravity as shown in Figure 10.

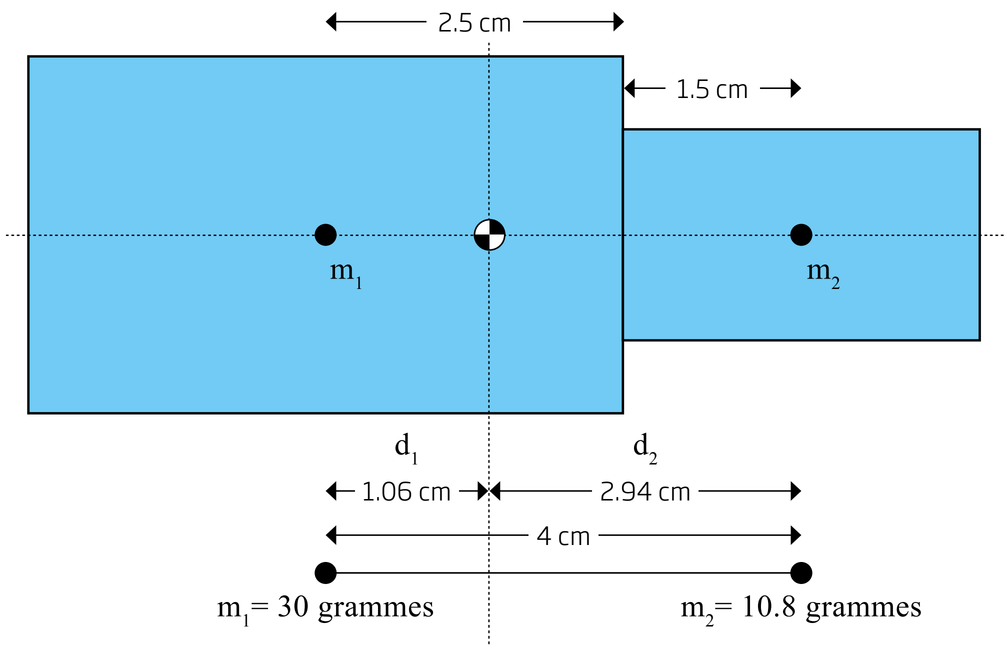


Figure 10 Balanced moments on the composite shape.

You can check the result above by cutting out a scale model of the shape from card as balancing it on a pin through a small hole made at the centre of gravity location. (Or by allowing the shape to hang from several corners in turn and marking a vertical line on the shape for each case – the lines will intersect at the centre of gravity.)

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