

# 5 Working with shape and space



## Technique sheet

Problem-based learning resources

### How do I find areas?

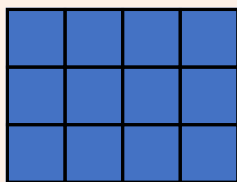
The space covered up by a flat surface is measured in square units. Most commonly square meters (m<sup>2</sup>) and square centimetres (cm<sup>2</sup>). You need to find how many of these square units fit inside the thing you are measuring. If the space is irregular, like a town on a map, or a jet engine fan blade, then you can fit squares inside and count them. For shapes made up from rectangles and triangles and for circles, you can use a set of rules for calculating the area.

### How do I work out areas for shapes made up from rectangles and triangles?

You will always need two dimensions. They may be referred to as length, width or depth but you can always call them base (b) and height (h). The height is always at right angles to the base. It is often called the perpendicular height.

#### Rectangle

$$\text{Area} = 4 \times 3 = 12\text{cm}^2$$

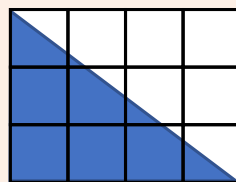


Base = 4cm

Height = 3cm

#### Right Triangle

$$\text{Area} = \frac{1}{2} \times 4 \times 3 = 6\text{cm}^2$$

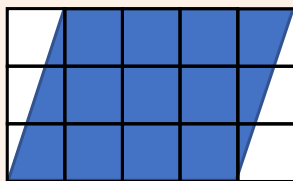


Base = 4cm

Height = 3cm

#### Parallelogram

$$\text{Area} = 4 \times 3 = 12\text{cm}^2$$

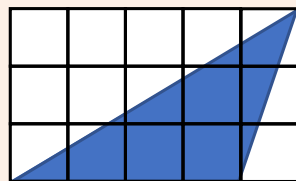


Base = 4cm

Height = 3cm

#### Triangle

$$\text{Area} = \frac{1}{2} \times 4 \times 3 = 6\text{cm}^2$$

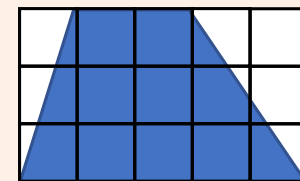


Base = 4cm

Height = 3cm

#### Trapezium

$$\begin{aligned} \text{Area} &= \text{bottom triangle} + \text{top triangle} \\ &= \frac{1}{2} \times 3 \times (2 + 5) = 10.5\text{cm}^2 \\ \text{Base } b &= 2\text{cm} \end{aligned}$$



Base a = 5cm

Height = 3cm

Rectangles & parallelograms  $A = bh$

Triangles  $A = \frac{1}{2} bh$

Trapezia  $A = \frac{1}{2} ah + \frac{1}{2} bh = \frac{1}{2} h(a + b)$

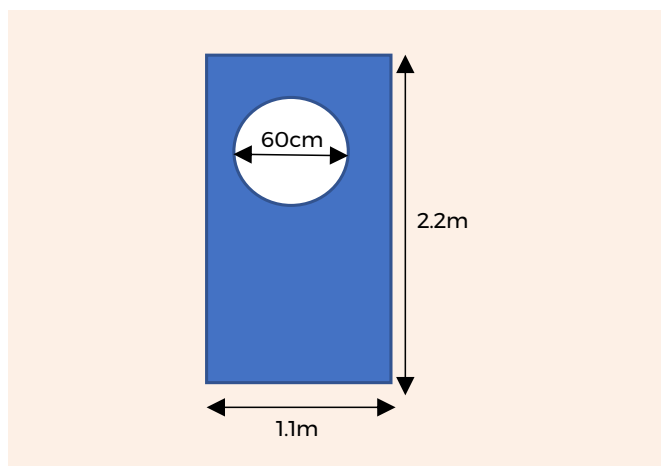
## How do I work out the Area and Circumference of a circle?

If you measure the perimeter of a circle (called the circumference), and divide that by the length of the diameter, then you always get exactly the same number: slightly more than 3. This number is given the special name pi and the symbol  $\pi$ .

For practical purposes the value  $\pi = 3.14$  is used. But scientific calculators have a  $\pi$  button which gives a very accurate value. This means that the circumference (C) is always  $\pi$  times the diameter (d).  $C = \pi d$ . The diameter is twice as long as the radius, so this is often written as  $C = 2\pi r$ . The area of a circle is  $\pi$  times the radius squared  $A = \pi r^2$ .

## How do I work out the Painted Area of the Door?

To work out the painted area of this door with a circular window, you need to work out the area of the rectangular door and take away the area of the circular window. The units are mixed, so you should make them consistent. Also, the formula uses the length of the radius and the length of the diameter is shown.  $60\text{cm} = 0.6\text{m}$ . So,  $r=0.3\text{m}$ .



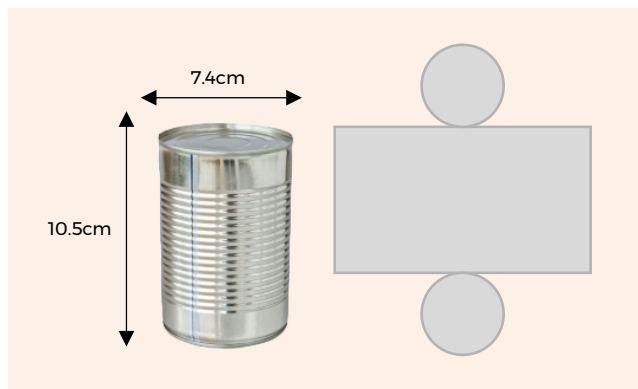
The area of the window is  $A = \pi r^2 = \pi \times 0.3^2 = 0.28\text{m}^2$  (to 2 d.p.)

The area of the door is  $2.2 \times 1.1 = 2.42\text{m}^2$ . So, the painted area is  $2.42 - 0.28 = 2.14\text{m}^2$

## How do I find out surface areas

To work out the area that covers the surface of a solid shape, called the surface area, you need to break down the outside of the shape into a flat surface, called a net. Then work out the area of all its parts. The steel in a can unfolds to a rectangle and two circles.

The total surface area is the rectangle plus the area of the two circles. Always look for ways to split up your shape into simpler shapes.



The rectangle area is the circumference of the circle

The circle area is  $\pi \times \text{radius}^2$

The surface area of the can the areas of 2 circles + the area of the rectangle.

## How do I calculate volumes of different solid shapes?

For volumes you need three dimensions, at right angles to each other. Now you need to know how many cube units will fit inside.

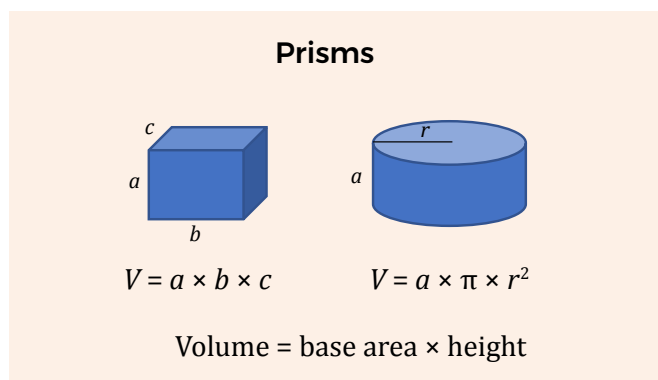
For irregular shapes we need ad hoc methods. You can work out your own volume by seeing how much the level of the water in your bath rises when you get in.

For regular shapes you can use a set of formulae. But first you have to decide what type of shape you are dealing with. There are three basic categories: prisms, pyramids and spheres. From these you can add and subtract when you have shapes made of up these types.

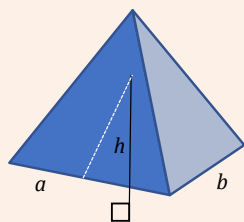
Prisms have a constant cross sectional area. Box shapes – cuboids, and cylinders are prisms.

Pyramids and cones have a base and extend to a single point.

To work out the volume of prisms and pyramids you need to know the base area and the length/height/depth, i.e. the other dimension at right angles to the base.



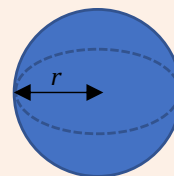
## Pyramid



$$V = \frac{1}{3} \times a \times b \times h$$

$$\text{Volume} = \frac{1}{3} \times \text{base area} \times \text{height}$$

## Sphere



$$V = \frac{4}{3} \times r^3$$

$$\text{Volume} = \frac{4}{3} \times \text{radius cubed}$$



### Check yourself

You should be able to answer these questions easily after reading this sheet.

1. How can you work out the area of a rectangle?
2. Give two examples of cuboid boxes.
3. What measure do you need to work out the volume of a sphere?



### Taking it further

These activities will deepen your understanding of this topic.

A neonatal incubator designed in India is made from a single cardboard cut out, which is the net of the solid incubator. Investigate the relationship between the area of the card and the volume produced in simple shapes like ordinary cuboid boxes. Why might it be useful to know the surface area of the incubator?

