# Study of Vibrations in JCB Dieselmax Engine

## JCB Power Systems Limited

### Mechanical Engineering

**INTRODUCTION**

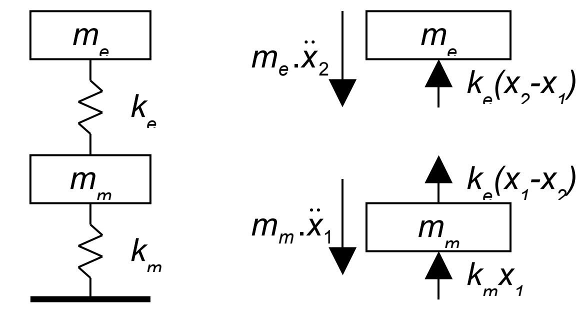
UK manufacturer JCB's construction machinery can be found on construction sites around the world. JCB is, in fact, the world's third largest producer of construction machinery. In recent years, JCB has started designing, developing and manufacturing of their own range of diesel engines. The JCB Dieselmax (named after a modified version of the mass produced digger engine that was used to power the JCB Dieselmax LSR car to a world land speed record of 350mph in August 2006) is used to power the majority of the JCB construction range. Typical applications include the backhoe loader and telescopic handler.


The JCB Dieselmax engine series is a 4 stroke, 4 cylinder range, with power outputs ranging from 63 kW to 120 kW.

**STUDY OF VIBRATIONS IN JCB ENGINE**

As well as considering the power requirements for a given application, the vibration of the system should be considered. The engine installation in a machine must ensure that the natural frequency of the system (the frequency at which a system will naturally oscillate when set into motion) is not equal to a typical excitation frequency. By "system" here, we mean the engine installed on a rubber mount. In this case, resonance may occur which will lead to an excessive vibration amplitude and will result in system failure.

A typical installation sees the engine placed in a machine the total mass of which ( $m_m$ ) is 2500 kg and the tyres act as a spring with effective stiffness ( $k_m$ ) of 500 kN/m. The engine is supported on rubber mounts with an effective stiffness ( $k_e$ ) of 100 kN/m. The engine mass ( $m_e$ ) is 500 kg. With this information, we can find the natural frequencies of this system.




Representation Free Body Diagram

The free body diagram illustrates the forces acting on each mass in accordance with Newton's Law of Motion which states that:

$$\text{Force} = \text{Mass} \times \text{Acceleration}$$

The forces here are occurring due to the stiffness of the tyres and rubber mounts, and hence follow Hooke's Law which states that:

$$\text{Force} = \text{Stiffness Coefficient} \times \text{Displacement}$$

For the system described here, it should be assumed that the two masses will move independently, by different amounts and with different speeds and accelerations, hence we denote the displacement of the machine mass by ' $x_1$ ' and the displacement of the engine mass by ' $x_2$ ' from the equilibrium position. Also, $x_1$ and $x_2$ are defined to be positive downward.

These forces must be in equilibrium and hence the equations of motion can be represented as:

$$m_m\ddot{x}_1 = -k_m x_1 - k_e(x_1 - x_2) \quad \ldots (1a)$$

$$m_e\ddot{x}_2 = -k_e(x_2 - x_1) \quad \ldots (1b)$$

or:

$$m_m\ddot{x}_1 + k_m x_1 + k_e(x_1 - x_2) = 0 \quad \ldots (2a)$$

$$m_e\ddot{x}_2 + k_e(x_2 - x_1) = 0 \quad \ldots (2b)$$

These two simultaneous equations can be represented as a matrix equation:

$$\begin{bmatrix} m_m & 0 \\ 0 & m_e \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_m + k_e & -k_e \\ -k_e & k_e \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad \ldots (3)$$

To solve this system of equations, we will use the method of substitution. Since the motion is vibrational, displacement $x$ depends on angular frequency $\omega$ and time $t$, and an appropriate substitution can be assumed as follows:

$$x = Xe^{j\omega t} \quad \ldots (4)$$

Here $j$ is a complex number such that $j = \sqrt{-1}$. Note that we now use $X$ instead of $x$ to denote the displacement component within the substitution to avoid confusion with the original displacement.

Differentiating equation (4) twice, we get:

$$\dot{x} = j\omega X e^{j\omega t} \quad \ldots (5a)$$

$$\ddot{x} = -\omega^2 X e^{j\omega t} \quad \ldots (5b)$$

Substituting these values back in equation (3), the matrix equation now becomes:

$$\begin{bmatrix} m_m & 0 \\ 0 & m_e \end{bmatrix} - \omega^2 e^{j\omega t} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} + \begin{bmatrix} k_m + k_e & -k_e \\ -k_e & k_e \end{bmatrix} e^{j\omega t} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Performing matrix addition, we get:

$$\begin{bmatrix} -\omega^2 m_m + k_m + k_e & -k_e \\ -k_e & -\omega^2 m_e + k_e \end{bmatrix} e^{j\omega t} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad \ldots (6)$$

Now, equation (6) will be true only if $X_1$ and $X_2$ both are zero, or if

$$\det \begin{bmatrix} -\omega^2 m_m + k_m + k_e & -k_e \\ -k_e & -\omega^2 m_e + k_e \end{bmatrix} = 0 \quad \ldots (7)$$

If $X_1$ and $X_2$ are both zero at the same time, this leads to the trivial solution for which there is no displacement at all. Thus, we are interested in the case where the determinant given in equation (7) is zero. On expanding the determinant, we get, progressively:

$$\left(-\omega^2 m_m + k_m + k_e\right)\left(-\omega^2 m_e + k_e\right) - k_e^2 = 0$$

$$\omega^4 m_m m_e - \omega^2 m_m k_e - \omega^2 m_e k_m + k_e k_m - \omega^2 m_e k_e + k_e^2 - k_e^2 = 0$$

$$\omega^4 m_m m_e + \omega^2\left(-m_m k_e - m_e k_m - m_e k_e\right) + k_e k_m = 0 \quad \ldots (8)$$

Equation (8) is actually a quadratic equation in $\omega^2$ (if a normal quadratic equation has the form $ax^2 + bx + c = 0$ and $x = \omega^2$ then the equation would be $a\omega^4 + b\omega^2 + c = 0$ which is what we have here). We can find the roots of such an equation using the standard formula for the solution of quadratics:

$$\omega^2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \ldots (9)$$

In our case, we have

$$a = m_m m_e = 2500 \times 500 = 1.25 \times 10^6$$

$$b = \left(-m_m k_e - m_e k_m - m_e k_e\right)$$

$$= (-2500 \times 100 \times 10^3 - 500 \times 500 \times 10^3 - 500 \times 100 \times 10^3)$$

$$= \left(-25 \times 10^7 - 25 \times 10^7 - 5 \times 10^7\right)$$

$$= 55 \times 10^7 = -5.5 \times 10^8$$

$$c = k_e k_m = 5 \times 10^{10}$$

Substituting these values in equation (8), we get:

$$\left(1.25 \times 10^6\right)\omega^4 + \left(-5.5 \times 10^8\right)\omega^2 + 5 \times 10^{10} = 0$$

Solving this equation using the quadratic formula (9), we find:

$\omega^2$ = 311.6515 and 128.3485 (rad/s)$^2$

Considering positive square roots only, we get:

$\omega$ = 17.6537 and 11.3291 rad/s.

Now, to find the natural frequency, we use the relationship

$$\omega = 2\pi f \quad \ldots (10)$$

where $f$ is the natural frequency of the system measured in Hz. Hence,

$f$ = 2.8097 Hz and 1.8031 Hz.

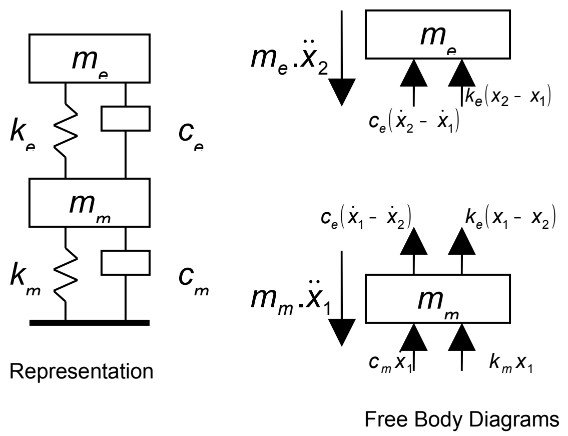
An ideal spring will carry on oscillating forever once set in motion unless an additional force is applied. In reality the tyres and engine mounts are spring-damper combinations. A damper provides a force proportional to the velocity of the oscillation; this dissipates energy and brings the system effectively to rest over time.

**EXTENSION ACTIVITY – 1:**

Find the natural frequencies of a system where the total mass of the machine ($m_m$) is 2000 kg, the tyres' effective stiffness ($k_m$) is 450 kN/m, the effective stiffness of the rubber mounts ($k_e$) is 250 kN/m and the engine mass ($m_e$) is 500 kg.

**EXTENSION ACTIVITY – 2:**

Show why we don't consider the dampers for the purpose of hand calculations. The diagram for this extension activity would be:



**Extension Activity – 3:**

What would be the effect on the natural frequencies obtained on page-2 in the solved example if:

(a) Engine mass is doubled

(b) Machine mass is doubled

(c) Both spring stiffnesses are doubled

(d) Both spring stiffnesses are halved

If one natural frequency of the system was targeted to be 5Hz, what would the other natural frequency be? Assume that the only factor we can change is the engine support mounts, i.e. the effective stiffness of the rubber mounts ($k_e$). Explain why this is practically unfeasible.

(*Hint: Use a back substitution process in the calculations discussed on pages 1 & 2 throughout.*)

**Where to find more**


1. *Engineering Mathematics*, Fifth Edition, John Bird, 2007, published by Elsevier Ltd.
2. *Internal Combustion Engine Fundamentals*, John Heywood, 1988, published by McGraw Hill
3. *Introduction to Internal Combustion Engines (2nd edition)*, Richard Stone, 1992, published by Macmillan
4. *Theory and Problems of Mechanical Vibrations: Schaum's Outline Series*, W.W. Setto, 1964, published by Schaum
5. http://auto.howstuffworks.com/diesel.htm




**Alan Curtis – Research Engineer**

Alan studied Mechanical Engineering at Loughborough University and graduated in 2006. Since then, Alan has worked for JCB Power Systems working on the design and specification of engines to comply with future emissions legislation. Alan devotes a generous portion of his time to supporting younger engineers at various stages of their education.

## Information for Teachers

The teachers should have some knowledge of

- Newton's laws of motion and Hooke's law of elasticity
- How to solve simultaneous equations using the matrix method followed by the substitution method that involves first and second derivatives
- How to expand a determinant
- Finding roots of a quadratic equation using the quadratic formula

## Topics covered from "Mathematics for Engineering"

- Topic 1: Mathematical Models in Engineering
- Topic 4: Functions
- Topic 5: Geometry
- Topic 6: Differentiation and Integration
- Topic 7: Linear Algebra and Algebraic Processes

## Learning Outcomes

- LO 01: Understand the idea of mathematical modelling
- LO 04: Understand the mathematical structure of a range of functions and be familiar with their graphs
- LO 07: Understand the methods of linear algebra and know how to use algebraic processes
- LO 09: Construct rigorous mathematical arguments and proofs in engineering context
- LO 10: Comprehend translations of common realistic engineering contexts into mathematics

## Assessment Criteria

- AC 1.1: State assumptions made in establishing a specific mathematical model
- AC 4.2: Analyse functions represented by polynomial equations
- AC 7.2: Use matrices to solve two simultaneous equations in two unknowns
- AC 9.1: Use precise statements, logical deduction and inference
- AC 9.2: Manipulate mathematical expressions
- AC 9.3: Construct extended arguments to handle substantial problems
- AC 10.1: Read critically and comprehend longer mathematical arguments or examples of applications.

## Links to other Units of The Advanced Diploma in Engineering

- Unit-1: Investigating Engineering Business and the Environment
- Unit-3: Selection and Application of Engineering Materials
- Unit-4: Instrumentation and Control Engineering
- Unit-5: Maintaining Engineering Plant, Equipment and Systems
- Unit-6: Investigating Modern Manufacturing Techniques used in Engineering
- Unit-7: Innovative Design and Enterprise
- Unit-8: Mathematical Techniques and Applications for Engineers
- Unit-9: Principles and Application of Engineering Science

## Answers to extension activities

**EA1:** 3.50 Hz and 1.72 Hz

**EA2:** The reason we don't consider them is because the final expressions for the determinant of the matrix contains terms in $\omega^4$, $\omega^3$, $\omega^2$ and $\omega$ and cannot therefore be solved as a quadratic equation.

**EA3:** (a) 2.59 Hz and 1.38 Hz, (b) 2.44 Hz and 1.47 Hz, (c) 3.97 Hz and 2.55 Hz, (d) 1.99 Hz and 1.28 Hz. With 5 Hz as one of the two frequencies, the second frequency must be 2.01 Hz (back substitution). For this frequency, the stiffness of the engine mounts would have to be 394522 N/m which is four times the current stiffness, and hence probably unattainable in practice.