The Mathematics of Escalators on the London Underground

Transport for London

Mechanical Engineering

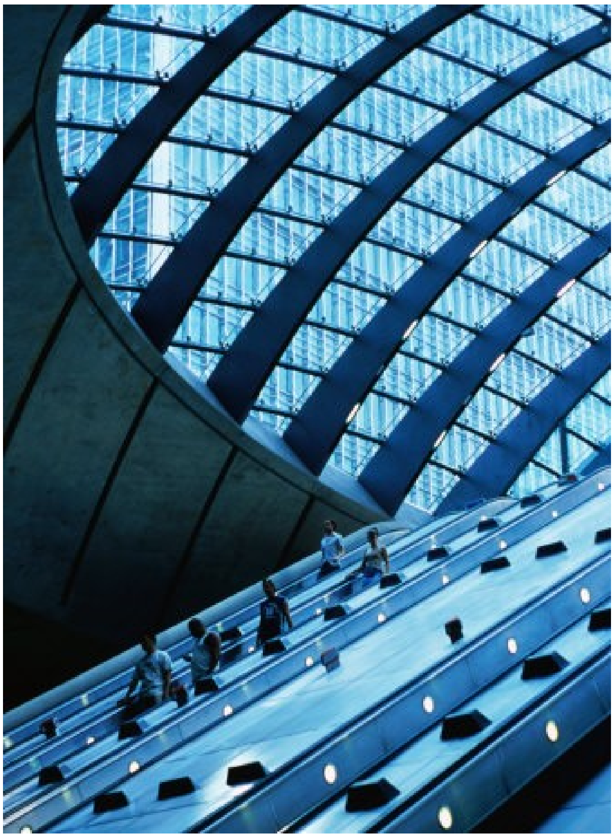

Figure-1: Escalators at Canary Wharf Station

Some Simple Calculations of Scale

4 million people use the London Underground every day. In the morning rush, around 20,000 people per hour travel down the 23 escalators at Canary Wharf Underground station. That's more than 300 people every minute.

$$\frac{20000 \text{ people per hour}}{60 \text{ minutes per hour}} = 333 \text{ people per minute}$$

Escalators on London Underground travel at a standard 0.75 m/sec on a standard 30° incline. An escalator step is 1 m wide and 400 mm (0.4 m) from front to back so a fresh step rolls out at the top of the escalator every 0.533 seconds.

$$\frac{0.4\,\text{m}}{0.75\,\text{m/sec}} = 0.533 \text{ sec}$$

If every step carries one person then the maximum capacity of the escalator is approximately 6,750 people per hour.

$$\frac{3600 \text{ sec/hr}}{0.533 \text{ sec/step}} = 6754 \text{ steps and people}$$

Consider an escalator that is 15 m along the incline. A step will take 20 seconds to travel that distance and, including 5 flat steps at the top and 4 at the bottom, there will be around 46 steps available for people to occupy at any one time. 37 people would take 20 seconds to rise a height of 7.5 m in the vertical plane.

$$\sin(30) = \frac{\text{opposite}}{\text{hypoteneuse}} = \frac{\text{height}}{15}\text{m}$$

$$\text{height} = 15 \times \sin(30) = 7.5\,\text{m}$$

If the average person has a mass of 75 kg, then the power required to lift 37 people would be about 10 kW.

$$\text{power} = \frac{\text{mass} \times g \times \text{height}}{\text{time}} = \frac{75 \times 9.81 \times 7.5 \times 37}{20} = 10.21\,\text{kW}$$

Here, g is the acceleration due to gravity. In addition to the mass of the person, the step they are standing on has a mass of 35 kg and the section of chain driving it a further 10 kg. But for every step travelling up on an escalator, there is another travelling down, so the effect of this additional mass is neglected. But of course, the escalator can accommodate almost 50% more people walking alongside those standing. Allowing 2 kW more for friction, and assuming a motor efficiency of 90% this 15m escalator typically consumes about 20 kW of electrical power.

How Green are Escalators?

How much CO_2 gets emitted for a single person on a 15 m escalator? 1 kW-hr of coal-fired electricity typically produces 0.9 kg of CO_2. Thus, the kW-hr per person on a single trip is:

$$\frac{(10.21 + 2)}{37} \times \frac{20}{3600} = 0.00183\,\text{kW-hr}$$

$$0.00183 \times 0.9 = 0.00165\,\text{kg} = 1.65\,\text{g}$$

Thus, 1.65 g of CO_2 is being emitted per person per trip.

How does this compare with the carbon emissions from travel on the underground trains themselves? This is commonly taken as 68 g of CO_2 per passenger kilometre. The equivalent emissions on the escalator are:

$$\frac{1000}{15} \times 1.65 \approx 110\,\text{g}$$

So, 110 g of CO_2 is being emitted per passenger kilometre going uphill. Going downhill the figure drops to 18 g (the figure required to overcome friction and motor inefficiency alone). Both of these figures are better than the emissions from a standard family car that emits 200 g of CO_2 per passenger kilometre with only one vehicle occupant. But the escalator emissions are 162% of the train emissions. **So why have escalators at all?**

Firstly, only the very fittest people could walk up the 100 or so stairs found at the typical deep tube station on the Underground. Also, even if everyone was super-fit, the number of people able to access, say Waterloo station in the morning rush, would drop from 51,000 to something like 34,000 assuming that a person can travel upstairs at a speed of 0.5 m/sec.

$$\frac{0.5}{0.75} \times 51{,}000 = 34{,}000$$

Replicated around the whole tube network, assuming that 3,000,000 people access the Tube network by escalator, this means a million less people a day could access the Tube network.

$$3{,}000{,}000 - \left(\frac{0.5}{0.75} \times 3{,}000{,}000 \right) = 1{,}000{,}000$$

If those million people drove only 5 km in a car, they would emit perhaps

$$1{,}000{,}000 \times 0.2 \times 5 = 1{,}000 \text{ tonnes } CO_2.$$

Whereas, 5 km by underground would emit

$$1{,}000{,}000 \times 0.068 \times 5 = 340 \text{ tonnes } CO_2.$$

This saves 640 tonnes of CO_2 for those 5 km. That's a good economics.

A 15 m *up* escalator, running for the typical 20 hours service each day on the London Underground, produces 220 kg of CO_2.

$$(10.2 + 2) \times 0.9 \times 20 \approx 220 \text{kg} = 0.22 \text{ tonnes } CO_2.$$

The one going down produces 36 kg of CO_2. With 418 escalators on the network, this means approximately 54 tonnes of CO_2 being released everyday by the escalators each day.

$$\frac{(220 + 36)}{2} \times 418 = 53.5 \text{ tonnes } CO_2.$$

Extension Activity – 1:

There are many hidden assumptions in this economic argument. What are they?

Applications of Trigonometry and of Differentiation

We know that passenger flows in and out of stations peak in the morning and evening rush hours and slow right down at other times. Even on the busiest stations, there will be times when the escalator is operating with nobody on it. How about we introduce an automated system that only operates the escalator when there is somebody there to ride it? This would clearly save a lot of CO_2. But it's not easy.

Extension Activity – 2:

If people approach a self-starting escalator from both ends simultaneously – will it go up or down? Try to think of ways to avoid this problem.

The common standard for escalators calls for acceleration levels on start-up or stop to be less than 1 m/sec^2 along the incline. Any more and there is significant risk of people being knocked off their feet. How is that measured?

Accelerometers (piezo-electric devices that sense acceleration) are fitted to an escalator step to measure vertical and horizontal acceleration as shown below

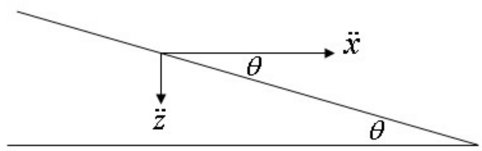

Figure-2: Horizontal and Vertical Components of Acceleration

The acceleration along the incline is therefore

$$\text{acceleration} = \ddot{x}\cos\theta + \ddot{z}\sin\theta$$

That acceleration could be integrated to yield velocity along the incline. Data of this type is shown in Figure – 3. The ideal velocity time history is shown with a dotted line. The electro-mechanical performance of an escalator is less than ideal so the actual velocity time history is something different.

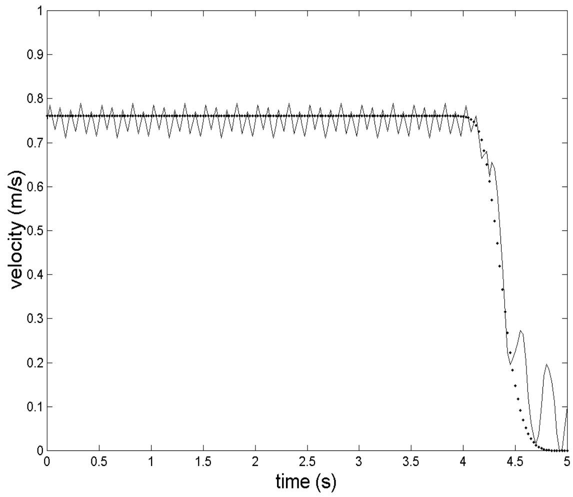

Figure-3: Velocity vs. Time

Applications of Percentages

Escalators are important to maximizing passenger flows on the Underground network. Therefore, their *availability* is measured.

$$\text{availability} = \frac{\text{running time}}{\text{planned running time}}$$

Escalators in service are very reliable, giving 100% availability. But if a hand rail slips, and takes 6 hours to repair? What will availability drop to if the reporting period for availability is 4 weeks?

$$100 - \left(\frac{6}{4 \times 7 \times 20} \right) \times 100 = 98.93\%$$

Where to find more

1. *Basic Engineering Mathematics*, John Bird, 2007, published by Elsevier Ltd.

2. *Engineering Mathematics*, Fifth Edition, John Bird, 2007, published by Elsevier Ltd.

3. http://science.howstuffworks.com/escalator1.htm

Tony Miller – Engineer on London Underground

Studied Physics before becoming an engineer in the power industry. Then he transferred to railways working first on rolling stock before moving into station machinery such as lifts and escalators.

INFORMATION FOR TEACHERS

The teachers should have some knowledge of

- Manipulating and using trigonometric functions (ratios in right-angled triangle)
- How to calculate percentages
- How to plot the graphs of simple functions using excel or other resources

TOPICS COVERED FROM "MATHEMATICS FOR ENGINEERING"

- Topic 4: Functions
- Topic 5: Geometry
- Topic 6: Differentiation and Integration

LEARNING OUTCOMES

- LO 03: Understand the use of trigonometry to model situations involving oscillations
- LO 04: Understand the mathematical structure of a range of functions and be familiar with their graphs
- LO 06: Know how to use differentiation and integration in the context of engineering analysis and problem solving
- LO 09: Construct rigorous mathematical arguments and proofs in engineering context
- LO 10: Comprehend translations of common realistic engineering contexts into mathematics

ASSESSMENT CRITERIA

- AC 3.1: Solve problems in engineering requiring knowledge of trigonometric functions
- AC 4.1: Identify and describe functions and their graphs
- AC 6.1: Calculate the rate of change of a function
- AC 6.3: Find definite and indefinite integrals of functions
- AC 9.2: Manipulate mathematical expressions
- AC 10.1: Read critically and comprehend longer mathematical arguments or examples of applications.

LINKS TO OTHER UNITS OF THE ADVANCED DIPLOMA IN ENGINEERING

- Unit-1: Investigating Engineering Business and the Environment
- Unit-3: Selection and Application of Engineering Materials
- Unit-5: Maintaining Engineering Plant, Equipment and Systems
- Unit-6: Investigating Modern Manufacturing Techniques used in Engineering
- Unit-7: Innovative Design and Enterprise
- Unit-8: Mathematical Techniques and Applications for Engineers
- Unit-9: Principles and Application of Engineering Science