Underground Storage Tank

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Chemical, Mechanical and Manufacturing Engineering, Metal Fabrication

Introduction

Storage of liquid petroleum products presents a threat to public health and the environment. Liquid petroleum products include fuel, such as petrol, diesel, oil, kerosene, etc. They are stored in Underground Storage Tanks (USTs). A tank is called an Underground Storage Tank if it has at least ten percent of its combined volume underground. It does not have to be completely underground. Such tanks are found at service stations, connected to boilers / steam generators, or connected to emergency generators.

Earlier USTs were made of bare steel, which was not protected from corrosion. Corrosion usually causes tanks to leak slowly. If an underground petroleum tank is more than 15 years old and not protected against rusting, the potential for leakage is much greater. Newer tanks and piping can leak too if they were not installed properly. Even a small fuel leak of one drop per second can result in the release of 200 litres of fuel into the groundwater over the period of one year. At low levels of pollution, water may smell or taste pure, yet be contaminated enough to harm human health.

In the case of petrol, once released from a tank, it sinks through unsaturated soil and, because it is less dense than water, floats on the surface of the water table. It can move so rapidly through surface layers that it will pollute the groundwater before the leak or spill has been noticed. Moreover, most components of petrol are fairly volatile and hence readily become a vapour at a relatively low temperature. This also presents risk of fire and explosion when vapours from leaking tanks can travel through sewer lines and soils into buildings through their basements or any other underground structures. So, preventing tank spills and leaks is very important to avoid the risks of water-contamination and explosion.

A metal-based UST, when installed underground, looks like:

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A typical UST can be constructed from one of the following materials:

- fibreglass-reinforced plastic
- steel with approved cathodic protection
- steel-fibreglass-reinforced-plastic composite
- metal without additional corrosion protection if the lack of such protection is viable.

USTs come in several different shapes, of which circular and elliptical are the most common.

Mathematical Modelling

An engineer at a chemical process plant needs to design a temporary storage facility for corrosive chemicals. Health and Safety requirements specify that, in the event of a leak in the storage facility, the chemicals must not escape beyond the immediate surroundings. The engineer therefore decides to use a standard tank of elliptical cross-section, partly buried in the ground, to a depth equal to the maximum height of fluid to be stored. Given a maximum volume of V of liquid chemicals to be stored, a tank of length, L, and semi-major axes, a and b (short side vertical), to what depth, d_{Final}, should the tank be buried?

Consider the cross-sectional view of the tank shown in the following figure:

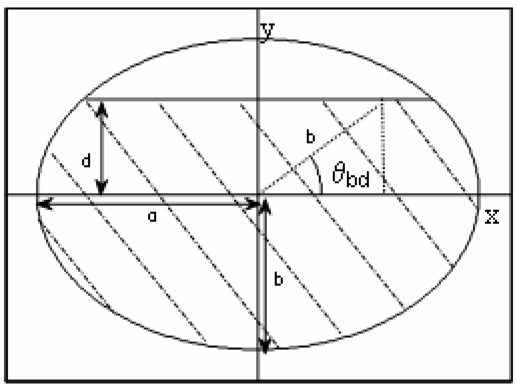

Figure-1: Cross-Sectional View of Elliptical Tank

The cross-sectional area occupied by liquid can be found from:

$$A = \frac{\text{Volume of Chemical, } V}{\text{Length of Tank, } L} \quad \ldots(1)$$

Since,

$$\text{Area of Ellipse} = \pi\, a\, b \quad \ldots(2)$$

this same area can also be given by (the use of integration to find the area between the curves)

$$A = \frac{1}{2}\pi\, a b + 2\int_0^d x\, dy \quad \ldots(3)$$

The standard equation of ellipse is given by

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$$

So

$$x^2 = a^2\left(1 - \left[\frac{y}{b}\right]^2\right)$$

or

$$x = a\sqrt{\left(1 - \left[\frac{y}{b}\right]^2\right)} \quad \ldots(4)$$

Substitute (4) into (3) to get the area of liquid as:

$$A = \frac{1}{2}\pi\, a b + 2\int_0^d a\sqrt{\left(1 - \left[\frac{y}{b}\right]^2\right)}dy \quad \ldots(5)$$

Now make the substitutions

$$y = b\sin(\theta) \quad \ldots(6a)$$

$$dy = b\cos(\theta)d\theta \quad \ldots(6b)$$

Using these substitutions, the integration limits will be converted as follows:

When $y = 0, \quad \theta = 0$ …(7a)

and when $y = d, \quad \theta = \theta_{bd}\,(say)$ …(7b)

θ_{bd} is related to b and d by combining (6a) and (7b) as follows:

$$d = b\sin(\theta_{bd}) \quad \ldots(8)$$

Substituting equations (6) and (7) into (5) we get:

$$A = \frac{1}{2}\pi\, a b + 2ab\int_0^{\theta_{bd}} \sqrt{\left(1 - \left[\frac{b\sin\theta}{b}\right]^2\right)}\cos\theta\, d\theta$$

$$A = \frac{1}{2}\pi\, a b + 2ab\int_0^{\theta_{bd}} \sqrt{\left(1 - \sin^2\theta\right)}\cos\theta\, d\theta$$

$$A = \frac{1}{2}\pi\, a b + 2ab\int_0^{\theta_{bd}} \cos^2\theta\, d\theta$$

$$A = \frac{1}{2}\pi\, a b + 2ab\int_0^{\theta_{bd}} \left(\frac{\cos 2\theta + 1}{2}\right)d\theta \quad \ldots(9)$$

Perform the integration in equation (9) to find:

$$A = \frac{1}{2}\pi\, a b + ab\left[\frac{\sin 2\theta}{2} + \theta\right]_0^{\theta_{bd}}$$

$$A = \frac{1}{2}\pi\, a b + ab\left[\frac{\sin 2\theta_{bd}}{2} + \theta_{bd}\right] \quad \ldots(10)$$

Combining equation (1) and (10), we get

$$\frac{V}{L} = \frac{1}{2}\pi\, a b + ab\left[\frac{\sin 2\theta_{bd}}{2} + \theta_{bd}\right] \quad \ldots(11)$$

The only unknown in equation (11) is θ_{bd}. We can find this using either a numerical method (e.g. the Newton-Raphson or Bisection methods) or graphical method or using some inbuilt function in software such as MathCad, Matlab or Autograph.

When θ_{bd} is small enough, i.e. when tank is just over half full, we can also solve equation (11) using the approximation that $\sin(2\theta_{bd}) \approx 2\theta_{bd}$. In this case, we can write equation (11) as

$$\frac{V}{L} = \frac{1}{2}\pi\, a b + ab\left[\frac{2\theta_{bd}}{2} + \theta_{bd}\right]$$

Rearranging the terms for θ_{bd}, we get

$$\theta_{bd} = \left[\frac{V}{L} - \frac{1}{2}\pi\, a b\right] \times \frac{1}{2ab} \quad \ldots(12)$$

Afetr finding θ_{bd} either by equation (11) or (12), we can find d using either equation (8) or equation (13) as follows, respectively:

$$d = b\theta_{bd} \quad \ldots(13)$$

Finally, the required depth will be

$$d_{Final} = b + d \quad \ldots(14)$$

Example Data: Check the above model for the following values:

Volume of the chemical to be stored, V = 40 m^3;

Length of the tank, L = 10 m;

Major axis of the elliptic tank, a = 1.75 m;

Minor axis of the elliptic tank, b = 1.25 m.

Solution: Applying a numerical method, we get θ_{bd} = 0.1293 radians (7.41°) and hence d = 0.1616 m. This shows that the tank should be buried in the ground up to a depth of d_{Final} = 1.25 + 0.1616 = 1.4116 metres.

Where to find more

1. *Basic Engineering Mathematics*, John Bird, 2007, published by Elsevier Ltd.
2. *Engineering Mathematics*, Fifth Edition, John Bird, 2007, published by Elsevier Ltd.

Dr. Alan Stevens – Specialist in Mathematical Modelling and Simulation (Retired, Rolls-Royce)

Spent 35 years as an industrial mathematician in the Submarines division of Rolls-Royce, dealing primarily with heat transfer and fluid-flow behaviour of the nuclear reactors used to power the Royal Navy's submarines.

Information for Teachers

The teachers should have some knowledge of

- Facts about the geometry of an ellipse, e.g. area of the ellipse = πab
- Integration by substitution

Topics covered from "Mathematics for Engineering"

- Topic 1: Mathematical Models in Engineering
- Topic 5: Geometry
- Topic 6: Differentiation and Integration

Learning Outcomes

- LO 01: Understand the idea of mathematical modelling
- LO 03: Understand the use of trigonometry to model situations involving oscillations
- LO 05: Know how 2-D and 3-D coordinate geometry is used to describe lines, planes and conic sections within engineering design and analysis
- LO 06: Know how to use differentiation and integration in the context of engineering analysis and problem solving
- LO 09: Construct rigorous mathematical arguments and proofs in engineering context
- LO 10: Comprehend translations of common realistic engineering contexts into mathematics

Assessment Criteria

- AC 1.1: State assumptions made in establishing a specific mathematical model
- AC 1.2: Describe and use the modelling cycle
- AC 3.1: Solve problems in engineering requiring knowledge of trigonometric functions
- AC 5.1: Use equations of straight lines, circles, conic sections, and planes
- AC 6.3: Find definite and indefinite integrals of functions
- AC 6.4: Use integration to find areas and volumes
- AC 9.1: Use precise statements, logical deduction and inference
- AC 9.2: Manipulate mathematical expressions
- AC 9.3: Construct extended arguments to handle substantial problems
- AC 10.1: Read critically and comprehend longer mathematical arguments or examples of applications.

Links to other Units of The Advanced Diploma in Engineering

- Unit-5: Maintaining Engineering Plant, Equipment and Systems
- Unit-6: Investigating Modern Manufacturing Techniques used in Engineering
- Unit-7: Innovative Design and Enterprise
- Unit-8: Mathematical Techniques and Applications for Engineers
- Unit-9: Principles and Application of Engineering Science