

INTRODUCTION

Laing O'Rourke plc is the largest privately owned construction firm in the UK. It has offices in the UK, Germany, India, Australia and United Arab Emirates and over 31,000 employees worldwide. It has a truly integrated capability, allowing their people to take a holistic view of construction. It specialises in delivering ambitious and innovative construction projects that deliver to both clients and the communities who use them every day.

IMPORTANCE OF EXEMPLAR IN REAL LIFE

Structures such as buildings and bridges consist of a number of components such as beams, columns and foundations all of which act together to ensure that the loadings that the structure carries is safely transmitted to the supporting ground below.

Normally, the horizontal beams can be made from steel, timber or reinforced concrete and have a cross sectional shape that can be rectangular, T or I shape. The design of such beams can be complex but is essentially intended to ensure that the beam can safely carry the load it is intended to support. This will include its own self-weight, the weight of the structure it is supporting and what is often referred to as "live load", i.e. the weight of people and furnishings in buildings or the weight of road or rail traffic in bridges.

Examples of beams can be seen in figures 1 to 3:



Figure-1: Steel Beams



Figure-2: Bridge Beams



Figure-3: Reinforced Concrete Beams

In addition to the requirements for the beam to safely carry the intended design loads, there are other factors that have to be considered including assessing the likely deflection of the beam under load. If beams deflect excessively, then this can cause visual distress to the users of the building and can lead to damage of parts of the building including brittle partition dividers between rooms and services such as water and heating pipes and ductwork.

Beam design is carried out according to principles set out in Codes of Practice. Typically, the maximum deflection is limited to the beam's span length divided by 250. Hence, a 5m span beam can deflect as much as 20mm without adverse effect. Thus, in many situations it is necessary to calculate, using numerical methods, the actual beam deflection under the anticipated design load and compare this figure with the allowable value to see if the chosen beam section is adequate. Alternatively, it may be necessary to check the ability of a given beam to span between two supports and to carry a given load system before deflections become excessive.

SCENARIO

As a graduate engineer working in a structural engineer's office, you have been given the task of assessing the maximum deflection that will occur in the steel beams of a steel-framed structure and ensuring that they are not excessive. In this exemplar, you will apply standard deflection formulae to solve some typical beam deflection design problems. These formulae form the basis of the calculations that would be undertaken in real life for many routine design situations.

BEAM SUPPORTED AT BOTH ENDS WITH UNIFORM LOADING

A simply supported beam carrying a uniformly distributed load over its length is shown in Figure-4 below:

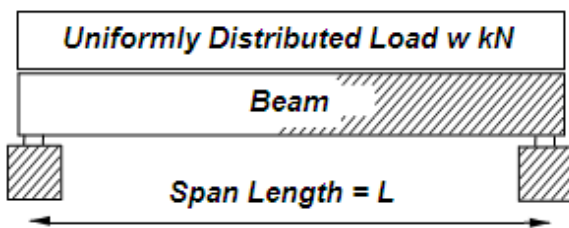


Figure-4: Simply supported beam with uniformly distributed load

The standard formula for finding deflection y at any given point x from one end, say from LHS, is given by:

$$y = \frac{wx(L-x)}{24EIL} [L^2 + x(L-x)] \dots (1)$$

where:

- E : Young's modulus of elasticity of the material from which the beam is fabricated (N/m^2)
- I : Area Moment of Inertia (m^4)
- L : Total length of the beam measured between centres of support (m)
- x : Some distance from one side of the beam, say from LHS (m)
- w : Load (N or kN)

Since the load is uniformly distributed throughout the Span Length L , it is obvious that the deflection will be maximum at the centre, i.e. when $x = L/2$. Substituting this value in equation (1), we get

$$\begin{aligned} y_{\max} &= \frac{w \frac{L}{2} \left(L - \frac{L}{2} \right)}{24EIL} \left[L^2 + \frac{L}{2} \left(L - \frac{L}{2} \right) \right] \\ &= \frac{w \frac{L^2}{4}}{24EIL} \left[L^2 + \frac{L^2}{4} \right] \\ &= \frac{wL}{24 \times 4 \times EI} \left[\frac{5L^2}{4} \right] \end{aligned}$$

$$= \frac{5}{24 \times 4 \times 4} \left[\frac{wL^3}{EI} \right]$$

Hence, the maximum deflection $y_{\max} = \delta$, say, at the centre is given by:

$$\delta = \frac{5}{384} \frac{wL^3}{EI} \dots (2)$$

To use the above formula, all the terms have to be converted to a consistent set of units. Units of force are usually expressed in kN and, as deflections are relatively small, it is convenient to convert all lengths to units of mm.

BEAM SUPPORTED AT BOTH ENDS WITH CONCENTRATED POINT LOAD BETWEEN THE SUPPORTS

A simply supported beam carrying a single concentrated point load at the centre is shown in Figure-5 below:

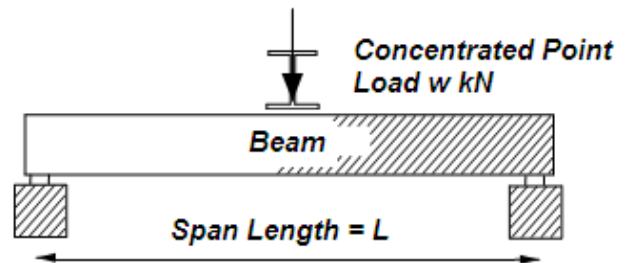


Figure-5: Simply supported beam with concentrated point load between the supports

This situation may occur when one beam is supporting another heavy localised beam in the building structure. In this case, the standard formula for finding deflection y between load and support points is given by:

$$y = \frac{wx}{48EI} [3L^2 - 4x^2] \dots (3)$$

where all the terms are as previously described. In this case again, the maximum deflection occurs at the mid-point, i.e. when $x = L/2$. Hence, substituting this value in equation (3), we get

$$\begin{aligned} y_{\max} &= \frac{w \frac{L}{2}}{48EI} \left[3L^2 - 4 \frac{L^2}{4} \right] \\ &= \frac{wL}{48 \times 2 \times EI} [2L^2] \end{aligned}$$

Hence, the maximum deflection $y_{\max} = \delta$ at the centre is given by:

$$\delta = \frac{1}{48} \frac{wL^3}{EI} \dots (4)$$

PLEASE NOTE: If the beam is carrying a combination of uniformly distributed load w_1 and a single point load at the centre w_2 , then the

maximum deflection at the centre can be obtained by combining the above two equations (2) and (4), i.e.

$$\delta = \frac{5}{384} \frac{w_1 L^3}{EI} + \frac{1}{48} \frac{w_2 L^3}{EI}, \text{ or}$$

$$\delta = \frac{L^3}{EI} \left[\frac{5w_1}{384} + \frac{w_2}{48} \right] \dots (5)$$

EXTENSION ACTIVITY – 1:

For a simply supported steel beam carrying a uniformly distributed load, calculate the unknown term (deflection, span length or loading) for each of the three design situations shown in the table below. In all cases take $E = 210 \text{ kN/mm}^2$ and $I = 45730 \text{ cm}^4$:

Deflection	Span L (m)	Distributed Load (kN/m)
To be calculated	10.0	15.0
Limited to span/250	10.0	Maximum load to be calculated
50mm	Maximum Span to be calculated	15.0

EXTENSION ACTIVITY – 2:

For a simply supported steel beam carrying a point load at mid-span calculate the unknown term (deflection, span length or loading) for each of the three design situations shown in the table below. In all cases take $E = 210 \text{ kN/mm}^2$ and $I = 45730 \text{ cm}^4$

Deflection	Span L (m)	Concentrated Point Load (kN)
To be calculated	10.0	100
Limited to span/250	10.0	Maximum load to be calculated
50mm	Maximum Span to be calculated	100

EXTENSION ACTIVITY – 3:

For a simply supported steel beam carrying a uniformly distributed load and a point load at mid-span calculate the unknown term (deflection, span length or loading) for each of the three design situations shown in the table below. In all cases take $E = 210 \text{ kN/mm}^2$ and $I = 45730 \text{ cm}^4$

Deflection	Span L (m)	Distributed Load (kN/m)	Concentrated Point Load (kN)
To be calculated	10.0	15.0	100
Limited to span/250	10.0	15.0	Maximum load to be calculated
50mm	Maximum Span to be calculated	15.0	100

EXTENSION ACTIVITY – 4:

Area Moment of Inertia, I , also known as Second Moment of Inertia, is a property of shape that is used to predict the resistance of beams to bending and deflection. The deflection of a beam under load depends not only on the load, but also on the geometry of the beam's cross-section. The product EI is known as the "**beam stiffness**" and it measures how strongly the beam resists deflection under bending moments. For a given material, i.e. E fixed, the beam stiffness can be maximised by maximising the value of I . This is done by using sectional shapes for which most of the sectional area is remote from the neutral axis (read below). For example, a beam of square cross-section is stiffer than a circular beam with the same area, since a circle has a larger proportion of the section near the neutral axis. A hollow square section is even stiffer. Taking this rationale still further leads to I-section beams and sandwich panels.

*(An **axis** in the cross section of a beam or shaft is a fictitious line where there are no longitudinal stresses or strain. If the section is symmetric and is not curved before the bend occurs then the **neutral axis** is at the geometric centroid. All fibers on one side of the neutral axis are in a state of tension, while those on the opposite side are in compression).*

Calculate the Area Moment of Inertia for different shapes like rectangular, circular, annular, I-shaped, etc. beams and record your findings. You may find reference 5 useful.

WHERE TO FIND MORE

1. *Basic Engineering Mathematics*, John Bird, 2007, published by Elsevier Ltd.
2. *Engineering Mathematics*, Fifth Edition, John Bird, 2007, published by Elsevier Ltd.
3. *Structural Mechanics*, Ray Hulse & Jack Cain, 2nd Edition, 2000, published by Palgrave.
4. http://www.engineersedge.com/beam_calc_menu.shtml
5. http://en.wikipedia.org/wiki/List_of_area_moments_of_inertia



Manos Adoniadis, Site Engineer – Laing O'Rourke

He says:

"I've recently graduated with a Masters in Civil Engineering from the University of Nottingham. As far back as I can remember I've been inquisitive and had a desire to know the answer to the question – How? This, and the way Civil Engineering can captivate and inspire drove me to pursue a career in engineering. I am currently working as a Site Engineer on a wastewater treatment works for Laing O'Rourke who sponsored me through my degree. Although not directly involved in the design of the project, as an Engineer on site, it's critical to have a good understanding of the design process, method and equations. This allows sound engineering decisions to be made on site with reference to standards of construction, components being used, and the construction process."

INFORMATION FOR TEACHERS

The teachers should have some knowledge of

- Terminology related to structural design and construction (e.g. bending moment, inertia, etc.);
- Substitution, manipulation and simplification of algebraic expressions

TOPICS COVERED FROM “MATHEMATICS FOR ENGINEERING”

- Topic 1: Mathematical Models in Engineering

LEARNING OUTCOMES

- LO 01: Understand the idea of mathematical modelling
- LO 09: Construct rigorous mathematical arguments and proofs in engineering context
- LO 10: Comprehend translations of common realistic engineering contexts into mathematics

ASSESSMENT CRITERIA

- AC 1.1: State assumptions made in establishing a specific mathematical model
- AC 1.2: Describe and use the modelling cycle
- AC 9.1: Use precise statements, logical deduction and inference
- AC 9.2: Manipulate mathematical expressions
- AC 9.3: Construct extended arguments to handle substantial problems
- AC 10.1: Read critically and comprehend longer mathematical arguments or examples of applications

LINKS TO OTHER UNITS OF THE ADVANCED DIPLOMA IN ENGINEERING

- Unit-1: Investigating Engineering Business and the Environment
- Unit-3: Selection and Application of Engineering Materials
- Unit-5: Maintaining Engineering Plant, Equipment and Systems
- Unit-6: Investigating Modern Manufacturing Techniques used in Engineering
- Unit-7: Innovative Design and Enterprise
- Unit-8: Mathematical Techniques and Applications for Engineers
- Unit-9: Principles and Application of Engineering Science

ANSWERS TO EXTENSION ACTIVITIES

EA1: 20.34mm, 29.501 kN/m, 12.522m

EA2: 21.69mm, 184.43kN, 13.210m

EA3: 42.03mm, 90.60kN, 10.510m

EA4: Please check.