Study of Dynamic Systems in JCB Construction Machinery

JCB Excavators Limited

Mechanical and Civil Engineering

INTRODUCTION

UK manufacturer JCB's construction machinery can be found on construction sites around the world. JCB is, in fact, the world's third largest producer of construction machinery. JCB's flagship product is the backhoe loader but their range of construction equipment products ranges from micro excavators through to articulated dump trucks.

A typical application for JCB machinery is within a quarry or on land which is being developed where the initial tasks are to clear waste material, dig foundations, etc. In this instance, JCB tracked excavators work in tandem with JCB articulated dump trucks to prepare the site efficiently.

Figure-1: A JCB machinery loading rubble from a construction site into a truck

PROBLEM STATEMENT

It is important to understand the dynamics of the scenario described here. When the dump truck is loaded, the additional mass causes a displacement, velocity and acceleration. We can calculate the effect on the system of this loading using the given properties of the system. This process is necessary to make sure that the displacement, velocity and acceleration caused during loading are not so large as to adversely affect the performance of the machine.

MATHEMATICAL MODELLING

With the application of maths, we can calculate the displacement, velocity and acceleration of the bed of the dump truck when a mass is dropped on it from the excavator.

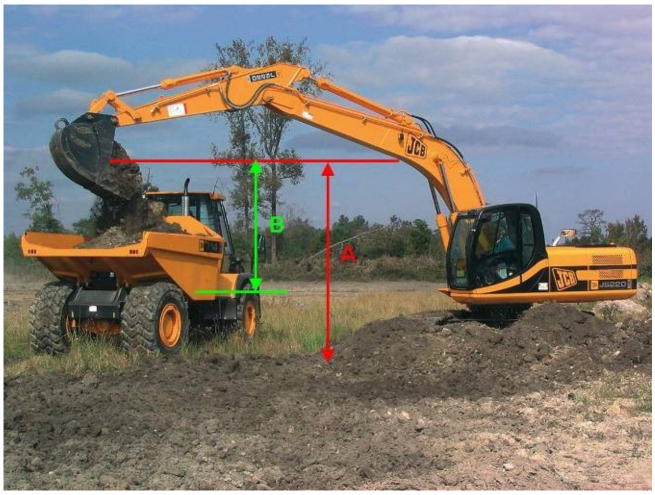


Figure-2: Showing distances *A* and *B*

In the diagram above, A represents the height to which the load is lifted before being released over the dump truck, and B represents the distance the load falls before it hits the bed of the dump truck. In this exemplar, let us assume these values to be A = 2.5 m and B = 1.5 m. The bed of the dump truck can be considered as a *mass-spring-damper system* where the mass of the truck bed, m_t = 2000 kg, the effective stiffness, k_t = 50,000 N/m and the effective damping, c_t = 3500 Ns/m. The mass of the load can be taken to be, m_l = 500 kg.

The static deflection (i.e. where the bed will rest when equilibrium is established) can be calculated by assuming that the additional weight due to the load is equal to the stiffness of the system k_t, multiplied by the deflection of the truck bed x_t, hence:

$$F = k_t x_t$$

$$\Rightarrow \; m_l g = k_t x_t$$

Rearranging the terms and substituting the known values, we get:

$$x_{t,static} = \frac{m_l g}{k_t}$$

$$= \frac{500 \times 9.81}{50{,}000}$$

$$= 0.0981\text{m} = 9.81\text{cm}$$

However, in order to calculate the maximum displacement of the system when the load is released onto the truck bed, we need to use the following equation involving the displacement of the bed of the dump truck (*please note that the derivation of this equation involves complex engineering dynamics which is beyond the scope of your syllabus, but* *the application of the result is all that is being considered here*):

$$x(t) = \frac{\sqrt{(v_t + \xi \omega x_{t,static})^2 + (x_{t,static}\omega_d)^2}}{\omega_d} e^{-\xi \omega t} \sin(\omega_d t) \quad \dots (1)$$

where:

$x(t)$	:	displacement of the truck at time t (m)
v_t	:	velocity of the truck bed on impact (m/sec)
ω	:	natural frequency of the truck bed (radians/sec)
ξ	:	damping coefficient
ω_d	:	damped natural frequency (radians/sec)

These variables are related through the following relationships:

$$\omega = \sqrt{\frac{k_t}{m_t}} \quad \dots (2a)$$

$$\xi = \frac{c_t}{2\omega m_t} \quad \dots (2b)$$

$$\omega_d = \omega(1 - \xi^2) \quad \dots (2c)$$

Substituting all the known values and using the calculated values as soon as they are available, we get:

ω = 5 radians/sec,

$\xi = 0.175$, and

$\omega_d = 4.847$ radians/sec.

Since the dropping of the load, and hence the impact on the bed of the truck is very quick, the transfer of momentum as the load strike the truck bed can be treated as an impulse.

The momentum of any moving object can be defined as:

$$\text{Momentum} = \text{mass} \times \text{velocity} \quad \dots (3)$$

In the case under discussion, the impulse sue to the load is therefore:

$$I_l = m_l \times v_l$$

Here, I_l represents the momentum of the load and v_l is the velocity of the load on impact which can be found by assuming that all the potential energy $PE = m \times g \times h$ lost as it falls through the height $h_l = B = 1.5$ is converted into kinetic energy $KE = \frac{1}{2}mv^2$, i.e.

$$PE = KE$$

$$\Rightarrow m_l \times g \times h_l = \frac{1}{2} m_l v_l^2$$

Rearranging the terms and substituting the known values, we get:

$$v_l = \sqrt{2 \times g \times h_l} = \sqrt{2 \times 9.81 \times 1.5} = 5.425 \text{ m/sec}$$

Furthermore, to find the velocity of the bed of the dump truck, we can apply the principle of conservation of momentum, which states that the momentum of the mass just before impact is completely transferred to the momentum of the object under collision. Using this principle for load and truck bed, we can write:

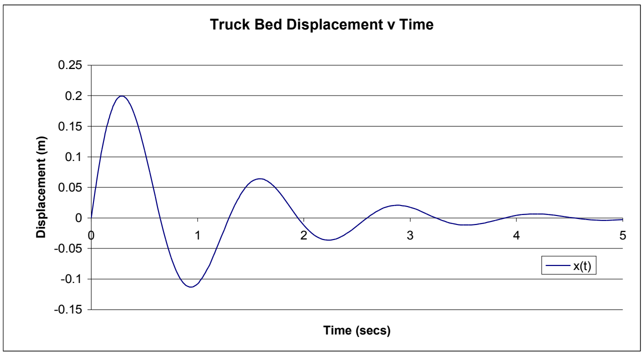
$$m_l v_l = m_{t,loaded} v_t \quad \dots (4)$$

Here, $m_{t,loaded} = m_t + m_l$, of course.

Rearranging the terms and substituting the known values, we get:

$$v_t = \frac{m_l v_l}{m_{t,loaded}} = \frac{500 \times 5.425}{2000 + 500} = \frac{500 \times 5.425}{2500} = 1.085 \text{ m/sec}$$

The maximum displacement of the truck bed can now be calculated using equation (1). The right-hand side of equation (1) contains the product of the sine function and an exponential function where both vary with time. A graph of the resulting variation of the displacement x_t with time is shown in the following figure:



From this graph, we can see that the maximum displacement occurs approximately at time t = 0.3sec and back substitution into equation (1) shows that x = 0.19 m at t = 0.3 sec. Visual inspection of the graph indicates that the maximum displacement is nearer x = 0.20 m than 0.19 m, so this is evidently not a very accurate estimate.

We can obtain a more accurate result by differentiating equation (1) and finding the time at which the displacement is a maximum and its value then. Substituting all the known and calculated values gives equation (1) the following form:

$$x_t(t) = 0.26066 e^{-\xi \omega t} \sin(\omega_d t) \quad \dots (5)$$

This equation is of the form:

$$y(t) = u(t).v(t)$$

This can be differentiated using the rule for the differentiation of the products, i.e.:

$$\frac{dy}{dt} = \frac{du}{dt} \cdot v + \frac{dv}{dt} \cdot u$$

Hence, differentiating equation (5), we get progressively:

$$\frac{dx}{dt} = 0.26066\left[\frac{d\left(e^{-\xi\omega t}\right)}{dt} \cdot \sin(\omega_d t) + \frac{d(\sin(\omega_d t))}{dt} \cdot e^{-\xi\omega t}\right]$$

$$\frac{dx}{dt} = 0.26066\left[\left(-\xi\omega\, e^{-\xi\omega t}\right) \cdot \sin(\omega_d t) + \omega_d.\cos(\omega_d t) \cdot e^{-\xi\omega t}\right]$$

$$\frac{dx}{dt} = 0.26066\; e^{-\xi\omega t}\left[-\xi\omega \cdot \sin(\omega_d t) + \omega_d.\cos(\omega_d t)\right]$$

… (6)

Now, $x(t)$ will be a maximum or a minimum when $\frac{dx}{dt} = 0$, i.e.

$$0.26066\; e^{-\xi\omega t}\left[-\xi\omega \cdot \sin(\omega_d t) + \omega_d.\cos(\omega_d t)\right] = 0$$

… (7)

Since $e^{-\xi\omega t} \neq 0$ for any t and ξ and ω are constants (already evaluated), we must have:

$$-\xi\,\omega\,\sin(\omega_d t) + \omega_d \cos(\omega_d t) = 0, \text{ or}$$

$$\tan(\omega_d t) = \frac{\omega_d}{\xi\omega}, \text{ or}$$

$$t = \frac{1}{\omega_d}\tan^{-1}\left(\frac{\omega_d}{\xi\omega}\right), \text{ or}$$

$$t = 0.29 \text{ sec}$$

At this time $t = 0.29$ sec, equation (1) gives $x =$ 0.1995 m which can be compared to the maximum point on the graph and we can therefore be confident that the figures are correct.

Note that, if we did not have the graph of $x(t)$ to inspect, then formally we would need to evaluate $\frac{d^2x}{dt^2}$ at $t = 0.29$ sec to determine whether we had found a maximum or a minimum of $x(t)$. For a maximum $\frac{d^2x}{dt^2}$ must be negative.

Note also that there are other solutions to equation (7) at

$$t = \frac{1}{\omega_d}\tan^{-1}\left(\frac{\omega_d}{\xi\omega}\right) + n \times \pi \quad \text{radians}$$

for n = 1, 2, 3… These correspond to the other maxima and minima of $x(t)$ which can clearly be seen in the graph.

Extension Activity – 1:

Calculate the acceleration of the bed of the truck at the point when the displacement is maximum.

Note: this can be done using the same differentiation principles used to calculate the velocity except here there is essentially double the number of calculations.

Extension Activity – 2:

Use a spreadsheet to calculate the values of displacement, velocity and acceleration from $t = 0$ to $t = 5$ seconds at an increment of 0.05 seconds.

Extension Activity – 3:

Explain why the following assumptions do not represent the real scenario:

1. All potential energy is converted into kinetic energy.
2. The mass hits the bed instantaneously in an evenly distributed manner.

Where to find more

1. *Basic Engineering Mathematics*, John Bird, 2007, published by Elsevier Ltd.
2. *Engineering Mathematics*, Fifth Edition, John Bird, 2007, published by Elsevier Ltd.

Alan Curtis – Research Engineer

Alan studied Mechanical Engineering at Loughborough University and graduated in 2006. Since then, Alan has worked for JCB Power Systems working on the design and specification of engines to comply with future emissions legislation. Alan devotes a generous portion of his time to supporting younger engineers at various stages of their education.

Information for Teachers

The teachers should have some knowledge of

- Newton's laws of motion and Hooke's law of stiffness
- Principle of conservation of momentum
- Product rule for differentiation
- Finding maximum values using first derivatives
- Plotting graphs using Excel or similar software

Topics covered from "Mathematics for Engineering"

- Topic 1: Mathematical Models in Engineering
- Topic 2: Models of Growth and Decay
- Topic 3: Models of Oscillations
- Topic 4: Functions
- Topic 6: Differentiation and Integration

Learning Outcomes

- LO 01: Understand the idea of mathematical modelling
- LO 02: Be familiar with a range of models of change, and growth and decay
- LO 03: Understand the use of trigonometry to model situations involving oscillations
- LO 04: Understand the mathematical structure of a range of functions and be familiar with their graphs
- LO 06: Know how to use differentiation and integration in the context of engineering analysis and problem solving
- LO 09: Construct rigorous mathematical arguments and proofs in engineering context
- LO 10: Comprehend translations of common realistic engineering contexts into mathematics

Assessment Criteria

- AC 1.1: State assumptions made in establishing a specific mathematical model
- AC 1.2: Describe and use the modelling cycle
- AC 2.1: Solve problems involving exponential growth and decay
- AC 3.1: Solve problems in engineering requiring knowledge of trigonometric functions
- AC 3.2: Relate trigonometrical expressions to situations involving oscillations
- AC 4.1: Identify and describe functions and their graphs
- AC 6.1: Calculate the rate of change of a function
- AC 6.2: Use derivatives to classify stationary points of a function of one variable
- AC 9.1: Use precise statements, logical deduction and inference
- AC 9.2: Manipulate mathematical expressions
- AC 9.3: Construct extended arguments to handle substantial problems
- AC 10.1: Read critically and comprehend longer mathematical arguments or examples of applications

Links to other Units of The Advanced Diploma in Engineering

- Unit-1: Investigating Engineering Business and the Environment
- Unit-3: Selection and Application of Engineering Materials
- Unit-4: Instrumentation and Control Engineering
- Unit-5: Maintaining Engineering Plant, Equipment and Systems
- Unit-6: Investigating Modern Manufacturing Techniques used in Engineering
- Unit-7: Innovative Design and Enterprise
- Unit-8: Mathematical Techniques and Applications for Engineers
- Unit-9: Principles and Application of Engineering Science

Answers to extension activities

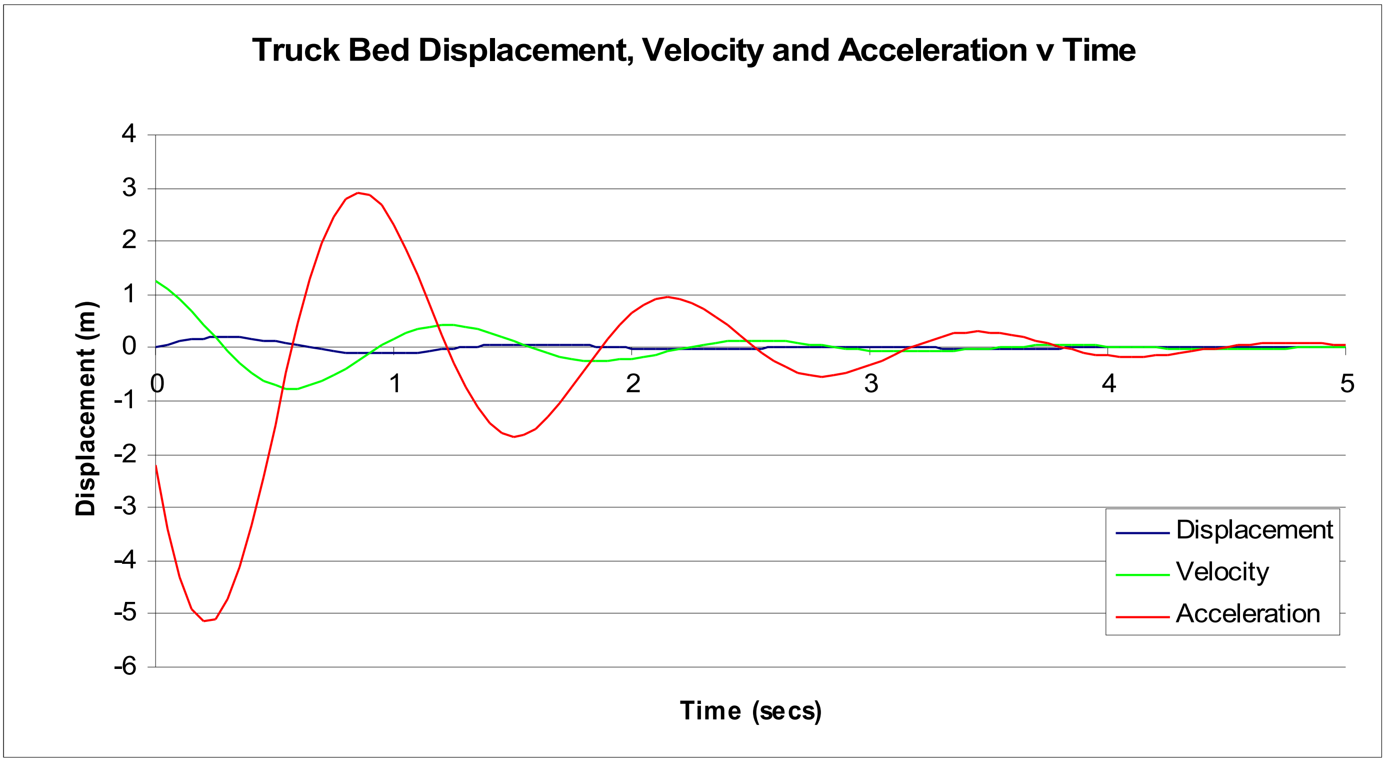
EA1:

$$\frac{d^2x}{dt^2} = 0.26066\, e^{-\xi \omega t}\left[\left(\xi^2\omega^2 - \omega_d^2\right)\sin(\omega_d t) - 2\xi\omega\omega_d\cos(\omega_d t)\right] \text{ or}$$

$$\frac{d^2x}{dt^2} = 0.26066\, e^{-\xi \omega t}\left[(-22.7277)\sin(\omega_d t) - (8.4823)\cos(\omega_d t)\right]$$

At $t = 0.3$ sec, we get $\frac{d^2x}{dt^2} = -4.7235 \text{ m/sec}^2$

EA2:



EA3:

1. Energy is lost to noise, heat, etc.
2. In reality, the mass will fall gradually (over an interval of time) and hit the bed gradually, and it is very likely that the impact and distribution of the load will be uneven due to the nature of the machinery being operated within the construction industry.