The Mathematics of Water Flow in Channels

Scenario

As a newly qualified graduate you have taken a year out to work for an overseas aid agency in a developing country. You have been assigned to work on a development project, the main aim of which is to provide water to a number of rural villages where clean water for drinking and for agricultural purposes is lacking. The project is intended to help the local population by improving general health through the provision of clean drinking water and to raise their income levels by encouraging agricultural developments.

It is intended to provide the water by diverting clean water from a natural reservoir in the nearby hills by passing it along a series of open channels that will be constructed for this purpose. Your task is to apply your engineering knowledge to ensure that the channels will be designed to have adequate capacity to carry the required flow of water over land that is generally flat and difficult to excavate.

Importance of Exemplar in Real Life

Civil engineers play a vital role in the planning, design and operation of water resource systems. In the specialist area of water resources engineering they design and manage urban and rural water supply together with storm-sewer systems. They study and develop flood forecasting techniques a vital area of engineering study as witnessed by the effects of severe flooding in parts of the UK in recent years.

In developing engineering solutions they apply the principles of fluid mechanics to water flowing in channels which include rivers and streams and also artificial channels, for example on irrigation schemes or water treatment works (See figures 1 and 2). Further applications include the design of sewage conduits, the management of waterways inclusing erosion and flood protection and environmental management, for example dealing with the effects of pollutants mixed with surface water.

Most of the above are examples of *Open Channel* Flow where water flows down an open channel with its top surface exposed to air. Civil engineers must ensure that such channels have enough capacity to carry the expected flow of water and to do this they will apply engineering solutions based on engineering theories modified to account for the physical characteristics of the flow channel including its shape and roughness characteristics.

These theories are equally applicable to studying and managing the flows down large rivers and the flows down small open channel irrigation systems used to provide sustainable solutions to the distribution of water to communities in remote and developing countries. Today modern sophisticated computer techniques enable the engineer to study such systems with increasing levels of complexity and sophistication.

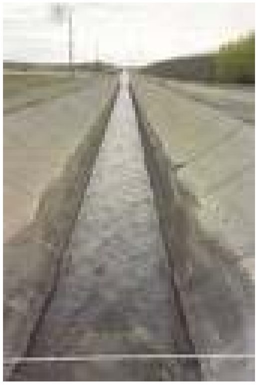

Figure 1: Open channel drainage system

Figure 2: Open channel irrigation system

<u>**Background Theory**</u>

(1) Uniform flow

'Uniform flow' occurs when the depth and the area of flow is constant with distance along the channel. With reference to figure 3, uniform flow occurs when the depth and area of flow at locations 1 and 2 are equal. It is unusual for flow in a river to be perfectly uniform because there is always some variation in depth or area along the river. However, in a long artificial channel it is more likely that conditions will be uniform, and it is often *assumed* that conditions are uniform as a simplification in design.

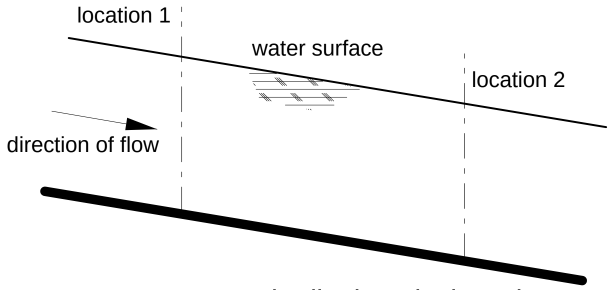

Figure 3: *Longitudinal vertical section*

(2) Channel shape properties

Channels can be all sorts of cross-sectional shapes. Channels in water treatment works often have simple rectangular shapes constructed in concrete (Fig 4(a)). Notice that this rectangular shape has three sides that are concrete surfaces. The fourth side of the rectangle is the water surface. Where a

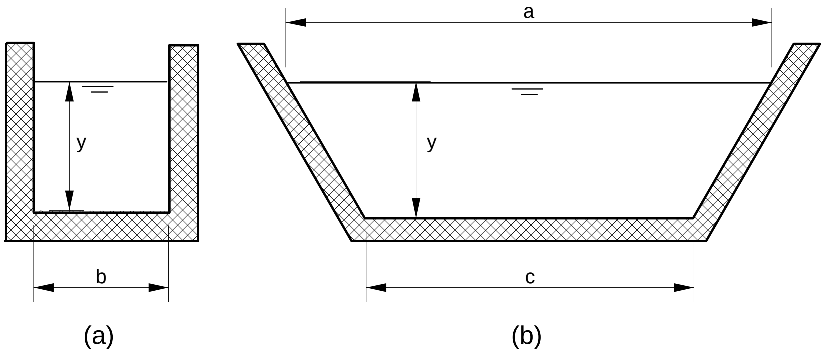

Figure 4: *Varying shapes of open channels*

channel is constructed from natural excavated material, the shape may be a trapezium (Fig 4(b)).
The difference between channel shapes is represented by a number of measurements or properties. We need to define four of these as:

y = depth

A = cross-sectional area (*y* × *b)* for the rectangle; or $y \times \frac{a+c}{2}$ for the trapezium

P = 'wetted perimeter': the length of contact between the water and the channel (*y* + *b* + *y)* for the rectangle)

R **=** 'hydraulic radius': the cross-sectional area divided by wetted perimeter, $= A/P$

(3) Slope

The slope of the channel (in the direction of flow) is the vertical 'fall' in the channel bed over a particular length divided by that length = *z*/*L* as shown in figure 5.

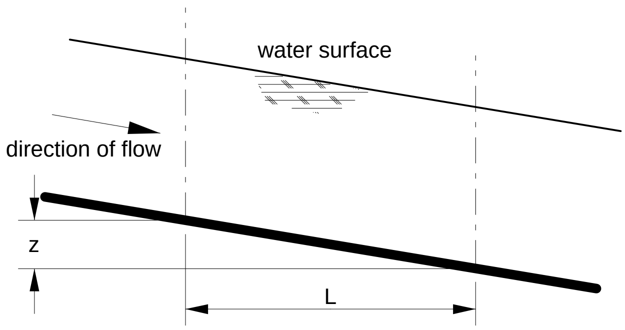

Figure 5: *Channel slope*

(4) Flow-rate

Flow-rate (*Q)* is the rate at which the water, in terms of volume, flows along a channel. It is expressed as volume divided by time, in units of m^3/s (cubic metres per second).

If we assume that flow in a channel is uniform, then the flow-rate is a function of:

- the area of flow in the channel (m^2)
- the hydraulic radius (m)
- the slope of the channel (in the direction of flow)
- the roughness of the channel material that is in contact with the water

Specifically, this can be represented in the **'Manning equation'**:

$$Q = \frac{A}{n} \times R^{\frac{2}{3}} \times S^{\frac{1}{2}}$$

where A = cross-sectional area
R = hydraulic radius
S = channel slope in the direction of flow
n = Manning coefficient of roughness

(5) Roughness

Roughness depends on the nature of the channel material. Some values of Manning's *n* are given below. As you can see, there is a range for concrete channels. The very smoothest concrete with a high quality finish has a value of about 0.012. Very rough concrete has a value of about 0.017. The values for rivers are higher, and of course there is a huge range. Laboratory channels with glass sides, used for research purposes and modelling, have a value of 0.010.

Material	***n (units: $s/m^{(1/3)}$)***
Concrete	0.012 – 0.017
River bed	0.020 – 0.080
Glass	0.010

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Questions

(1) You have been employed on the design of the irrigation scheme intended to provide water to a number of rural villages where clean water for drinking and for agricultural purposes is lacking. It is proposed to provide open 0.9m wide rectangular-section channels lined with concrete with the water flowing to a maximum depth of 1.2 m. It is required to have a capacity of 2.0 m^3/s (the highest flow-rate it is expected to carry). If the channel is constructed using rough finished concrete ($n = 0.017$), what slope (in the direction of flow) will be needed assuming that the flow is uniform? If the fall of the land is fairly shallow and the maximum fall of the channels for economic construction is restricted to 0.3m per 100 metres is this design adequate?

(2) Using a spreadsheet carry out an investigation into alternative designs that will meet the design specification. Investigate (a) the effect of widening or narrowing the channel and (b) the effect of providing a better standard of concrete finish.

(3) Investigate the effect of providing a trapezoidal shape of section as in figure 4(b). The bottom width of the section is 0.9m and the side slopes are 2 vertical to 1 horizontal and again it is proposed to use rough finished concrete. All other parameters are as in (1) above.

(4) Again, using a spreadsheet carry out an investigation into the effect of providing a trapezoidal shape of section with varying side slopes and varying roughness coefficients.

Where to find more

1. Melvyn Kay, *Practical Hydraulics,* 2nd edn, Spon 2006 (ISBN 978-0-203-02297-9)
2. Bird J, *Engineering Mathematics,* 5th edn, Elsevier, 2007 (ISBN 978-07506-8555-9)

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INFORMATION FOR TEACHERS

Teachers will need to understand and explain the theory outlined above and have knowledge of:

- ❑ Some principles of hydraulics and construction methods
- ❑ Simple Geometry

Topics covered from Mathematics for Engineers

- Topic 1: Mathematical Models in Engineering
- Topic 4: Functions

Learning Outcomes

- LO 01: Understand the idea of mathematical modelling
- LO 04: Understand the mathematical structure of a range of functions and be familiar with their graphs
- LO 09: Construct rigorous mathematical arguments and proofs in engineering context
- LO 10: Comprehend translations of common realistic engineering contexts into mathematics

Assessment Criteria

- AC 1.1: State assumptions made in establishing a specific mathematical model
- AC 1.2: Describe and use the modelling cycle
- AC 4.1: Identify and describe functions and their graphs
- AC 9.1: Use precise statements, logical deduction and inference
- AC 9.2: Manipulate mathematical expressions
- AC 9.3: Construct extended arguments to handle substantial problems
- AC 10.1: Read critically and comprehend longer mathematical arguments or examples of applications

Links to other units of the Advanced Diploma in Construction & The Built Environment

Unit 1 The Relationship of the Built Environment to the wider environment and community
Unit 3 Civil Engineering Construction

Solution to the Questions

1. Slope = 0.00439, fall = 0.439m per 100 metre – No, too steep

2. (a) increasing the width to 1.1m would reduce the fall to 0.244 per 100 metres (b) using a good quality of concrete finish (n=0.012) would reduce the fall to 0.219m per 100 metres

3. The fall would be 0.089m per 100 metres – indicating the beneficial effect of providing a trapezoidal section

4. Answers will vary but this exercise will enable the learner to investigate the mathematical relationships between all the variables in the problem

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