

## Scenario

You are working in a materials testing laboratory with specific responsibility for testing and monitoring the strength of concrete specimens as they are brought in for testing. Currently, your company has a contract for testing the concrete on a major development that is part of the 2012 Olympic Games construction work. The complexity of the project and its high profile means that strict quality control is necessary.

In this exercise you will consider some of the statistical issues that are relevant to the testing and control of concrete quality based on the testing of concrete cubes which are manufactured on site and then tested after, normally, 28 days in the laboratory.

## Importance of Exemplar in Real Life

Concrete as a construction material can be found in all forms of construction from the construction of housing, schools, hospitals, commercial buildings, industrial buildings, water infrastructure supplies, roads, railways and bridges. The commercial activity associated with concrete in construction amounts to about £4 billion and has significant social and economic benefits. When combined with steel reinforcing bars, in the form of reinforced concrete it provides one of the most versatile and economic forms of construction materials.

However, as with all construction materials, its quality can vary depending on the batching, mixing and placing processes and the standard of site supervision and control. Concrete for structural use is designed to satisfy the requirements of the structure for which it is intended. Considerations of *workability* (the ability to 'flow' when being placed as wet concrete) and *durability* (the ability to ensure that a structure remains serviceable throughout its lifetime) will govern the concrete mix design in terms of the minimum cement content per m<sup>3</sup> of concrete and the maximum water/cement ratio. The environmental exposure of the structure will dictate these requirements such that, for example, a lesser quality of concrete will be required if the structure is subject to dry conditions rather than wet conditions.

One of the most important considerations is that of the concrete strength. Concrete strength is usually related to its strength at 28 days based on the load that a standard cube of concrete ( figure 1) can sustain when tested in compression in a compression testing rig (figure 2). This load is converted to *stress* by dividing the load at failure by the surface area of the cube. The *strength* of the concrete is then quoted in terms of N/mm<sup>2</sup> and typical strengths will vary from as low as 20 N/mm<sup>2</sup> to more typically of the order of 30 N/mm<sup>2</sup> whilst, with modern concrete technology, strengths as high as 90 to 100 N/mm<sup>2</sup> are possible. Such high strengths have facilitated the construction of very tall buildings such that the very high loads and hence stresses at the lower floor levels can be sustained by this high strength concrete.



**Figure 1:** Moulds for making cubes



**Figure 2** Compression Testing Rig

## Background Theory

### (1) Assumed statistical behavior

When designing any form of reinforced concrete structure the designer will specify the strength of concrete that has been assumed in the design. However such an assumption will recognise the likely variability of concrete as a structural material and the designer will specify the concrete's design *characteristic strength*. This characteristic strength is based on statistical concepts and is the strength below which no more than 5% of all cubes tested from the chosen concrete mix will fall. Equally it can be expected that 95% of all cube samples will have strengths in excess of the design characteristic strength. This concept is illustrated in figure 3 which shows a histogram of cube strengths from a particular project and how these results will approximate to a *Normal Distribution* curve.

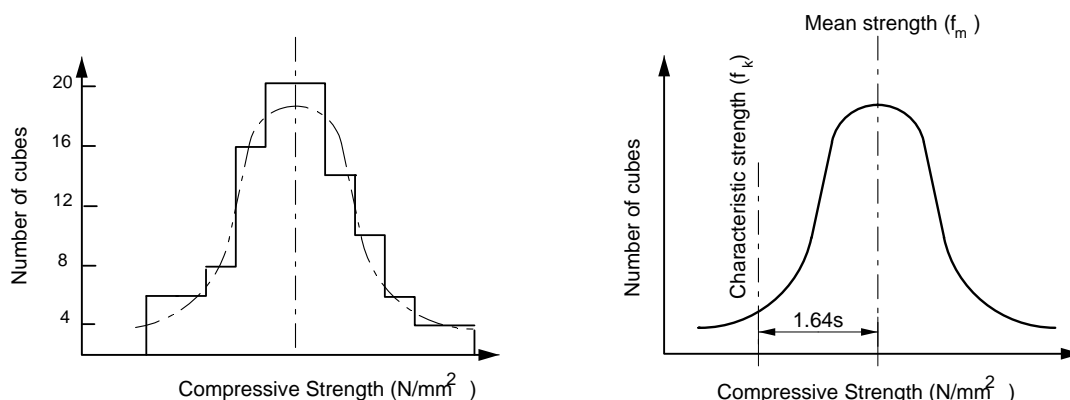


Figure 3: Histogram of cube compression strengths      Figure 4: Normal frequency distribution of strengths

Figure 4 shows the idealized Normal Distribution curve that is assumed in design and for ensuring quality control based on statistical procedures. The lower limit indicated on the diagram is the *characteristic strength* below which no more than 5% (1 in 20) of strength tests should fall. This is given by:  $f_k = f_m - 1.64s$  where  $f_k$  = characteristic strength,  $f_m$  = mean strength and  $s$  = standard deviation. The standard deviation is given by the standard formula:

$$s = \left[ \frac{\sum (f_c - f_m)^2}{n-1} \right]^{\frac{1}{2}} = \left[ \frac{n \sum (f_c)^2 - (\sum f_m)^2}{n-1} \right]^{\frac{1}{2}}$$

where  $f_m = (\sum f_c) / n$ ,  $n$  is the number of strength tests contributing to the sample and  $f_k$  is the compressive strength of each cube.

The relationship between characteristic and mean strength values accounts for variation in results of cube specimens and will reflect the method and control of manufacture, quality of constituents and the nature of the material.

### (2) Quality Control

During construction sets of cubes are taken from samples of concrete used on site and, normally, tested at 28 days after manufacture although earlier testing can be done, typically at 7 days, where rapid curing techniques have been adopted on site. For a large construction project where concrete pouring may take place over many days or weeks, a large set of test data will accumulate and each test result (based on an average of two cube tests) can be compared with the specified mean and characteristic values. Quality Control Standards might state, for example, that if three results in any forty sets of consecutive results fall below the characteristic value then there may be an issue with the concrete's quality control.

However, such a fairly simplistic approach can lead to quality issues only becoming identifiable after a large number of test results have been taken. A better predictor of issues is to develop and make use of a *Mean Strength (CUSUM)* chart. To plot such a chart, the *target mean strength* is deducted from each test result and these differences are summed to find a CUMulative SUM which is then plotted as

shown in figure 5. Thus, as more results become available, the plot can be continuously developed and changes in trends can be identified. For example, figure 5 shows that there has been a reduction in means strength subsequent to the testing of cube 55 and action should be taken to address this apparent fall in strength.

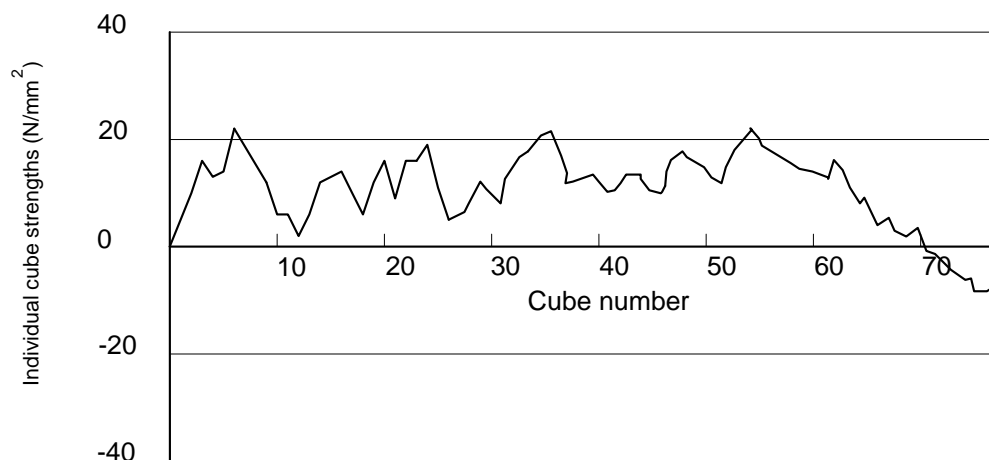


Figure 5: CUSUM Control chart for mean strength

In such circumstances, if there is an apparent fall in strength over  $n$  test results, the mean strength ( $f_{m(mean)}$ ) represented by these results can be estimated as:

$$f_{m(mean)} = f_m + \frac{\text{(change in CUSUM over } n \text{ results)}}{n}$$

where  $f_m$  is the *target mean* strength for design of the mix.

Such CUSUM charts can be plotted for individual test results, mean of sets of results or standard deviations.

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## Questions

**Example Data:** Calculate (a) the mean strength and (b) the standard deviation for the set of concrete cube results shown in the table below, then (c) plot the CUSUM chart for the complete set of results. (d) estimate the mean strength ( $f_{m(\text{mean})}$ ) given by the results taken between cube 18 and cube 31. The concrete's characteristic strength ( $f_k$ ) and target mean strength ( $f_m$ ) are 25 and 33 N/mm<sup>2</sup> respectively based on a design standard deviation of 5 N/mm<sup>2</sup>

DATA		DATA	
Cube No	$f_c$	Cube No	$f_c$
1	36.64	21	23.47
2	40.96	22	34.29
3	27.42	23	32.09
4	24.10	24	24.89
5	41.44	25	25.67
6	34.94	26	33.78
7	25.32	27	38.60
8	41.14	28	37.23
9	37.36	29	31.31
10	31.64	30	25.39
11	41.42	31	26.39
12	37.53	32	42.99
13	23.34	33	32.69
14	36.43	34	38.16
15	31.81	35	36.49
16	33.65	36	42.87
17	33.88	37	41.54
18	38.19	38	28.60
19	24.79	39	37.37
20	24.81	40	30.22

## Where to find more

1. Jackson N, Dhir RK, *Civil Engineering Materials*, 5<sup>th</sup> Edn, Palgrave, 1997 (ISBN 978-03336-3683-1)
2. BSEN 206-1, *Concrete specification, performance, production and conformity*. British Standards Institute 2000 (ISBN 0-580-36882-3)
3. Bird J, *Engineering Mathematics*, 5<sup>th</sup> edn, Elsevier, 2007 (ISBN 978-07506-8555-9)

## INFORMATION FOR TEACHERS

Teachers will need to understand and explain the theory outlined above and have knowledge of:

- ❑ Some understanding of concrete testing technology
- ❑ Statistical Methods

### Topics covered from Mathematics for Engineers

- Topic 1: Mathematical Models in Engineering
- Topic 8: Statistics and Probability

### Learning Outcomes

- LO 01: Understand the idea of mathematical modelling
- LO 08: Understand how to describe situations using statistics and use probability as a measure of likelihood
- LO 09: Construct rigorous mathematical arguments and proofs in engineering context
- LO 10: Comprehend translations of common realistic engineering contexts into mathematics

### Assessment Criteria

- AC 1.1: State assumptions made in establishing a specific mathematical model
- AC 1.2: Describe and use the modelling cycle
- AC 8.1: Summarise a set of data
- AC 8.2: Describe a random sample and how it might be taken
- AC 9.1: Use precise statements, logical deduction and inference
- AC 9.2: Manipulate mathematical expressions
- AC 9.3: Construct extended arguments to handle substantial problems
- AC 10.1: Read critically and comprehend longer mathematical arguments or examples of applications

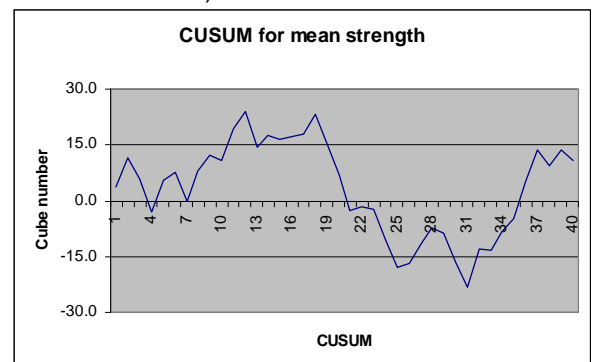
### Links to other units of the Advanced Diploma in Construction & The Built Environment

Unit 3 Civil Engineering Construction

Unit 29 Science and Materials in Construction and the Built Environment

### Solution to the Question (see calculations on the next sheet for CUSUM)

(a)  $33.3\text{N/mm}^2$ , (b)  $6.26\text{ N/mm}^2$  (c)  $29.7\text{ N/mm}^2$  (c)



Similar exercises can be developed to that indicated above using different data sets. Such data sets can be readily generated on a spreadsheet using a random number generation function.

Cont:-

DATA		CALCULATIONS FOR CUSUM	
Cube No	$f_c$	$f_c - f_m$	Cusum
(1)	(2)	(3) = (2)-33	
1	36.64	3.6	3.6
2	40.96	8.0	11.6
3	27.42	-5.6	6.0
4	24.10	-8.9	-2.9
5	41.44	8.4	5.6
6	34.94	1.9	7.5
7	25.32	-7.7	-0.2
8	41.14	8.1	7.9
9	37.36	4.4	12.3
10	31.64	-1.4	10.9
11	41.42	8.4	19.4
12	37.53	4.5	23.9
13	23.34	-9.7	14.2
14	36.43	3.4	17.7
15	31.81	-1.2	16.5
16	33.65	0.6	17.1
17	33.88	0.9	18.0
18	38.19	5.2	23.2
19	24.79	-8.2	15.0
20	24.81	-8.2	6.8
21	23.47	-9.5	-2.7
22	34.29	1.3	-1.4
23	32.09	-0.9	-2.4
24	24.89	-8.1	-10.5
25	25.67	-7.3	-17.8
26	33.78	0.8	-17.0
27	38.60	5.6	-11.4
28	37.23	4.2	-7.2
29	31.31	-1.7	-8.9
30	25.39	-7.6	-16.5
31	26.39	-6.6	-23.1
32	42.99	10.0	-13.1
33	32.69	-0.3	-13.4
34	38.16	5.2	-8.3
35	36.49	3.5	-4.8
36	42.87	9.9	5.1
37	41.54	8.5	13.7
38	28.60	-4.4	9.3
39	37.37	4.4	13.6
40	30.22	-2.8	10.8

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