

Scenario

As a new graduate you have gained employment as a graduate engineer working for a major contractor that employs 2000 staff and has an annual turnover of £600m. The company has sent you to work on a £12m highway development building a two mile by-pass around a small rural village that, for many years, has been blighted by heavy traffic passing through its narrow main street.

Your first task is to determine the alignment of the new road using information provided by the Consulting Engineering company that has designed the new road. To do this you will need to establish the position of the road centre-line and locate marker pegs at set intervals in the open fields around the village where the road will eventually be built. These marker pegs will be used as control points to ensure that the road is built in the correct position.

In this exemplar you will calculate the geometric coordinates of the road-centre line marker pegs

Importance of Exemplar in Real Life

In the UK we spend in excess of £ 6 billion per annum on improving and developing our highway infrastructure (government figures 2005/2006). This is a big business and there are many large Civil Engineering companies engaged in both designing and building of both major and minor highway schemes. Our highway network extends over a distance of nearly 400,000 kilometres!

As with all Civil Engineering projects, highway schemes are built to exacting standards to ensure that modern highways are capable of carrying modern heavy traffic loads and have anticipated life-spans that justify the massive public expenditure in this infrastructure work. For example major highways are designed to carry millions of vehicles per year, specified in terms of the number of standard axles which is a measure designed to average out the effects of different types and weights of vehicles. Typically they will be designed to have a life-span of between 20 and 40 years before major maintenance and upgrading is required.

Highways must also be designed and built such that they ensure comfort and safety to the road user and also are built in such a way that they make the most effective use of all available resources including materials, construction plant and labour. *Sustainability* issues may be high on the agenda and the road may have to be aligned to avoid, for example, areas of natural beauty or conservation areas.

Highway Engineers have to ensure that the alignment of the road in both plan and elevation meets current design standards and ensures that the road meets the many demands placed on it such as those outlined in the paragraph above. Highway alignment design is carried out according to principles set out in UK Standards such as those produced by the *Highway Agency*. (see <http://www.standardsforhighways.co.uk/dmrb/index.htm>.) and will be determined by requirements such as the design speed of the road, the minimum radius of bends and curves and the need to ensure that drivers can overtake safely with adequate visibility ahead.

Nowadays highway alignment is carried out using highly sophisticated computer programmes which can produce amazing visual graphics such that the design engineer can “walk through” the design and view the road on screen from a driver’s perspective. However, such programmes are based on mathematical techniques which can be demonstrated through the exercise described in this worksheet.

Photographs of the highway scheme covered by this exemplar can be seen in figures 1 to 4 below:

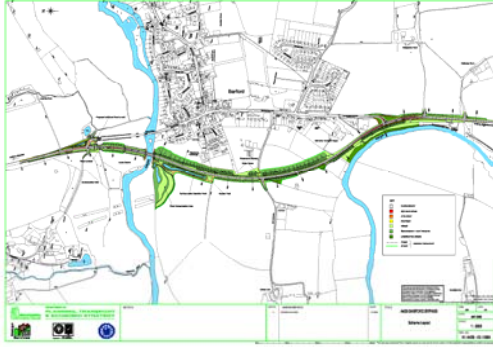


Figure 1: Road alignment as seen on a map



Figure 2 Road alignment as seen from the air



Figure 3: Road curving over a river



Figure 4 Road with junctions after construction

Background Theory

Horizontal curvature

The horizontal alignment of the road, as can be clearly seen in Figure 1, consists of a series of circular curves connected by straight lines. In practice between the straight lines and the circular curves will be a series of *transition curves* that will ensure that a driver, driving at speed, can leave a straight section of road and not be suddenly affected by the centrifugal forces that will be developed as the vehicle moves at a speed around the circular curve. However, for the purpose of this exercise we will neglect the transition curves and focus on the geometry of the straight sections and the circular curves only.

Figure 5 shows the intersection of two straight sections of road at the *Intersection Point I* and the *Tangent Points* T_1 and T_2 where the circular curve joins the two straight sections. The radius of the circular curve is R and it subtends an angle θ at the centre of the curve.

The following formula are relevant :

$$\text{Length of tangent from I to } T_1 \text{ (or } T_2) = R \tan \frac{\theta}{2} \quad (1)$$

$$\begin{aligned} \text{Length of curve from } T_1 \text{ to } T_2 &= R\theta \text{ where } \theta \text{ is measured in radians} \\ &= R\theta \frac{\pi}{180} \text{ where } \theta \text{ is measured in degrees} \end{aligned} \quad (2)$$

The distance of any point along the road is denoted by its **chainage**, being its distance, measured along the road centre line, from a chosen origin or start point of the road. *Chainage* refers to a technique of measurement where steel chains of 100 links were once used to measure distances. Although such equipment is no longer used the term *chainage* is still commonly used to denote points of distance measurement along a road centre-line. Hence if the location of the intersection point, I, is known then:

$$\text{Chainage of } T_1 = \text{chainage of I} - IT_1 = \text{chainage of I} - R \tan \frac{\theta}{2} \quad (3)$$

$$\text{Chainage of } T_2 = \text{chainage of } T_1 + \text{curve length } T_1 T_2 = \text{chainage of } T_1 + R \times \theta \frac{\pi}{180} \quad (4)$$

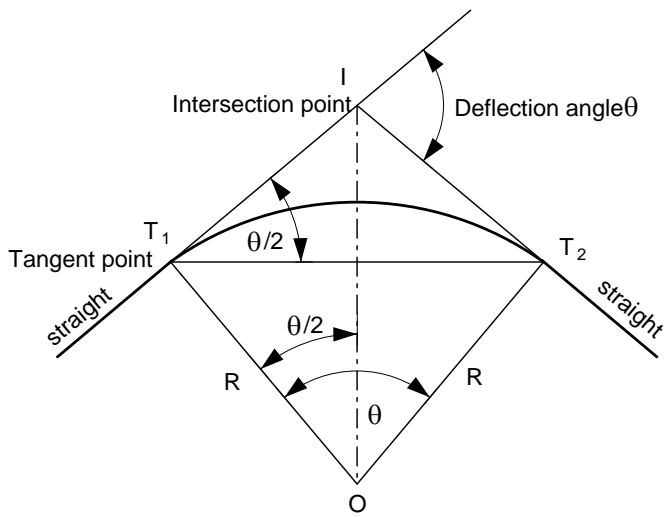


Figure 5: Geometry of Circular Curve

Figure 6: Engineer setting out Tangent Points

Providing that the intersection point is in an accessible position the site engineer will establish its position in the field from data established in the design office, will set up an electronic measuring instrument, (known as a theodolite or total station – see figure 6) at the intersection point and sight along the line I to T_1 . By measuring the calculated tangent distance from I to T_1 the position of the first tangent point can be established in the field. Similarly the second tangent point can be located by the same means. Normally, these points are physically located by the engineer hammering a wooden peg into the ground and accurately positioning a nail at the centre of the peg at the exact location of the tangent point. The measurement of distance can be done using a long tape but frequently electronic distance measuring techniques are used.

Locating the position of other points along the curve

Once the site engineer has located the tangent points, the theodolite or total station will be moved to one of the tangent points and a sighting taken to a point along the straight section to give an opening bearing. The intermediate points along the curve can be determined by turning off the *deflection angle* α and measuring the *chord length* T_1 to A as indicated in figure 7.

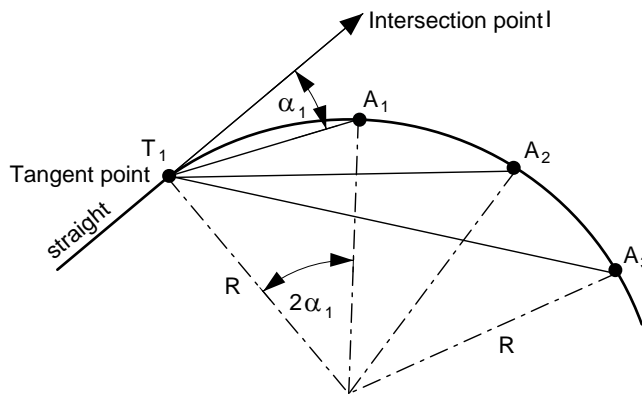


Figure 7: Locating intermediate points along the curve

For example, to locate the point A_1 the following formula are relevant :

If the angle $IT_1A_1 = \alpha_1$ then the angle subtended at the centre of the circle = $2\alpha_1$

Hence **arc length** T_1A_1 = $R \times 2\alpha_1 = 2R\alpha_1$ where α_1 is measured in radians

Or arc length T_1A_1 = $2R\alpha_1 \frac{\pi}{180}$ where α_1 is measured in degrees (5)

Hence α_1 is given by: $\alpha_1 = T_1A_1 \frac{180}{2\pi R}$ where in this equation T_1A_1 is the arc length (6)

$$\text{The chord length } T_1A_1 = 2R\sin\alpha_1 \quad (7)$$

If the chord length is no greater than $R/20$ then the length of the chord can be shown to be approximately the same as the length of the arc.

The site engineer will calculate the chord length and deflection angle for a series of chainage points along the curve and, using the theodolite or total station, will sight from T1 to a point on the previous straight section and then turn off the angle $180^\circ + \alpha_1$. By sighting down this line and measuring the chord length distance from T_1 to A_1 the location of the chainage point A_1 can be established. This can be repeated to find the position of other chainage points along the curve. Normally chainage points are established at fixed intervals of, say, 20 metres.

The above calculations illustrate one of the main ways that data is derived to set out highway alignments. However there are many variations on the above to cope with, for example, situations where the curve length is very large and it is not possible to establish its position from a single tangent point location or indeed where it is not even possible to gain access to the tangent point position.

The tutor should consult the references at the end of this exemplar to see how the geometry of the circle can be used to calculate curve setting out data in such circumstances and to derive appropriate exercises.

Question

Example Data: Calculate the setting out data for a circular curve of radius 400m connecting two straight sections of road with a deflection angle of 20° . The chainage of the intersection point is 2000m and centre-line pegs are to be located at 20m chainages.

Where to find more

1. Schofield W, Breach M, *Engineering Surveying*, 6th edn, Oxford: Butterworth-Heinemann
2. Bird J, *Engineering Mathematics*, 5th edn, Elsevier, 2007 (ISBN 978-07506-8555-9)
3. *Design Manual for Roads and Bridges*. See: <http://www.standardsforhighways.co.uk/dmrb/index.htm>.

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INFORMATION FOR TEACHERS

Teachers will need to understand and explain the theory outlined above and have knowledge of:

- ❑ Some terminology relating to highway construction
- ❑ Geometry and properties of the circle

Topics covered from Mathematics for Engineers

- Topic 1 Mathematical Models in Engineering
- Topic 5 Geometry

Learning Outcomes

- LO 01: Understand the idea of mathematical modelling
- LO 05: Know how 2-D and 3-D coordinate geometry is used to describe lines, planes and conic sections within engineering design and analysis
- LO 09: Construct rigorous mathematical arguments and proofs in engineering context
- LO 10: Comprehend translations of common realistic engineering contexts into mathematics

Assessment Criteria

- AC 1.1: State assumptions made in establishing a specific mathematical model
- AC 1.2: Describe and use the modelling cycle
- AC 5.2: Calculate distances
- AC 9.1: Use precise statements, logical deduction and inference
- AC 9.2: Manipulate mathematical expressions
- AC 9.3: Construct extended arguments to handle substantial problems
- AC 10.1: Read critically and comprehend longer mathematical arguments or examples of applications

Links to other units of the Advanced Diploma in Construction & The Built Environment

- Unit 2 Site Surveying
- Unit 3 Civil Engineering Construction
- Unit 6 Setting Out Processes

Solution to the Question

Radius	400			Chainage
Deflection angle	20		I	2000.000
Tangent length	145.588		T1	1854.412
Curve length	139.626		T2	1994.038
Point	Chainage	arc length	alpha	chord length
		metres	degrees	metres
T1	1854.412			
1	1860.000	5.588	0.400	5.588
2	1880.000	25.588	1.833	25.584
3	1900.000	45.588	3.265	45.563
4	1920.000	65.588	4.697	65.515
5	1940.000	85.588	6.130	85.425
6	1960.000	105.588	7.562	105.282
7	1980.000	125.588	8.995	125.073
T2	1994.038	139.626	10.000	138.919

Similar exercises can be developed to that indicated above; for example connecting two curves by a straight section with the second curve acting in the reverse direction to the first.

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