Dropped Load Assessment in Nuclear Industry

British Energy

Civil and Nuclear Engineering, Process Industry, Health and Well Being

INTRODUCTION

British Energy (BE) is part of EDF Energy. The combined enterprise is the UK's biggest producer of electricity and together provides power to a quarter of the country's population. It operates a number of nuclear power stations, coal and gas power stations, as well as combined heat and power plants and wind farms.

SCENARIO

As a civil engineer in the Civil Design Group of British Energy (BE), you are part of a team working with multiple engineering disciplines including a variety of design reviews, structural assessments and on-site inspections. The structural assessment involves the analysis of nuclear safety related civil structures for all BE nuclear power stations. The topic we are going to discuss in this exemplar is dropped load – which is one of the most important aspects in the impact assessment activities.

IMPORTANCE OF THE EXEMPLAR

In the nuclear power stations during normal station operation and maintenance procedures, there is a demand to move heavy loads using cranes and lifting beams. During load handling, there is a potential for the load being carried to be dropped. Dropped objects generally have relatively high velocities which can cause a great damage to the impacted structure. However, this damage is normally very local and includes yielding of materials, local buckling and, in some cases, local failure or penetration. An incident of this nature might directly or indirectly cause a fault which could lead to an increased risk of a radiological hazard.

Therefore, it is very significant to perform such structural integrity assessment of dropped load. A periodic safety review is needed to make sure that all the lifting equipments work properly under the norms of health and safety.

MATHEMATICAL MODEL

When an object impacts on a target with a velocity, the velocity of the impactor is changed significantly within a very short period of time. Since the impactor is released from a height, it also has a positive acceleration (gravitational acceleration in case of a freely falling object and forced acceleration in case of an object with some initial velocity). As a result of this impact, the positive acceleration changes to a very large negative acceleration. During this process, a significantly high compressive force is produced at the interface of the impactor and the target which can cause damage to the target and/or impactor depending on its strength. We have to make sure that these forces generated at the interface are within the desired limit in order to reduce the damage caused due to impact.

Assuming the weight of the impactor is W, this inertia force, F, can be then calculated as follow:

$$F = \frac{W}{g} a \; \dots (1)$$

where a is the acceleration of the impact and g is gravitational acceleration. Due to this inertia force, a dynamic stress and dynamic strain are produced in the target and the impactor. In theory, we should be able to calculate these forces using equation (1). But the velocity change during the impact is so short in time that the negative acceleration is impossible to be measured accurately; consequently the dynamic stress and strain are unlikely to be calculated on the basis of inertial force (equation 1).

Furthermore, an engineer is more interested in calculating the maximum dynamic stress and strain produced in the impacted body(s) so that he can compare it with the set safety norms.

To answer such an important question raised in nuclear industry, we will now discuss the energy conservation method which provides a sufficiently accurate approximation for these two important engineering variables.

In order to simplify the mathematical model, we will make some assumptions as follows:

1. the impactor is a rigid body (i.e. its deformation is significantly smaller in comparison to that taking place in the target)
2. the mass of the targeted body is neglected (i.e. its mass is much smaller than the impactor's mass)
3. during impact, all kinetic energy is converted into strain energy (other energy losses are so small as can be neglected)

For most engineering problems, the targeted structure can be regarded as an elastic system, like a spring-mass system, where the force F is linearly related to the deformation y in the target. It can be shown by the following equation:

$$F = ky \; \dots (2)$$

where k represents the spring elastic constant. This equation (2) is also known as Hooke's Law of Elasticity.

Consider the following simplified system shown in Figure-1.

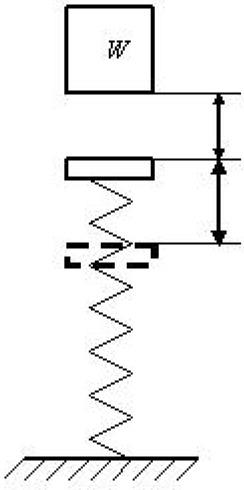

Figure-1: The simplified system

Here, an impactor (or load) of weight W is being dropped on an elastic target from a height h causing a dynamic impact force F_d and dynamic deformation y_d. Therefore, from equation (2), we can write:

$$F_d = ky_d \;\ldots\; (3)$$

Experiments show that for such a system (linear elastic system) under consideration, the elastic constant k will remain same under static loading circumstances as well, i.e.:

$$W = ky_s \;\ldots\; (4)$$

where y_s represents the static deformation caused by the static loading W.

Substituting this in equation (3), we get:

$$F_d = \frac{W}{y_s} y_d \;\ldots\; (5)$$

According to the energy conservation, all kinetic energy (E_{max}) of the impactor is converted into elastic strain energy (U_{max}):

$$E_{max} = U_{max} \;\ldots\; (6)$$

Since the impact is caused by a freely-dropped object on the target, the kinetic energy of the impactor in nothing but its initial potential energy. Therefore:

$$E_{max} = W(h + y_d) \;\ldots\; (7)$$

For any system under loading, the elastic strain energy is equal to half of the product of the force and the corresponding deformation. Thus, in case of dynamic loading, we have:

$$U_{max} = \frac{1}{2} F_d y_d \;\ldots\; (8)$$

This can quickly be visualised in the following figure 2:

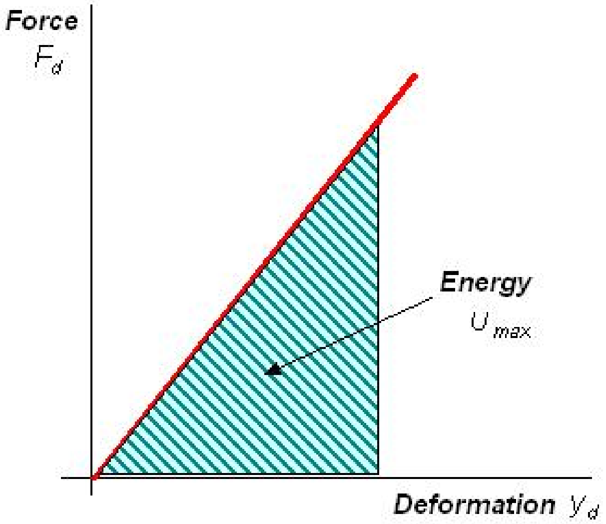

Figure 2: Graphical illustration of Elastic Strain Energy

Substituting the value of F_d from equation (5), we get

$$U_{max} = \frac{W}{2y_s} {y_d}^2 \;\ldots\; (9)$$

Finally, substituting equation (7) and (9) into equation (6), we have

$$W(h + y_d) = \frac{1}{2}\frac{W}{y_s} {y_d}^2 \;\ldots\; (10)$$

By rearranging the above equation, we get:

$$ {y_d}^2 - 2y_s y_d - 2hy_s = 0 \;\ldots\; (11)$$

which is a quadratic equation of the type

$$ax^2 + bx + c = 0 \;\ldots\; (12)$$

whose solution is given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \;\ldots\; (13)$$

On comparing equation (11) and (12), we can write the solution of equation (11) as follows:

$$y_d = \frac{-(-2y_s) \pm \sqrt{(-2y_s)^2 - 4 \times 1 \times (-2hy_s)}}{2 \times 1}$$

On simplification, we get:

$$y_d = y_s \pm \sqrt{{y_s}^2 + 2hy_s} \;\ldots\; (14)$$

In the current scenario, we have $y_s > 0$ and $h > 0$. Also, we are interested in the maximum dynamic y_d. Hence, considering only the positive sign, we successively get:

$$y_d = y_s + \sqrt{{y_s}^2 + 2hy_s}\text{ , or}$$

$$y_d = y_s\left(1+\sqrt{1+\frac{2h}{y_s}}\right)\text{, or}$$

$$y_d = \mu y_s \ \ldots (15)$$

where

$$\mu = \left(1+\sqrt{1+\frac{2h}{y_s}}\right) \ \ldots (16)$$

which is known as dynamic factor.

Substituting equation (15) in equation (5), we get:

$$F_d = \mu W \ \ldots (17)$$

For the case we are interested as illustrated in Figure 1, the stress (σ) can be expressed as applied force (F) divided by the cross section area (A),

$$\sigma = \frac{F}{A} \ \ldots (18)$$

Hence, we have $\sigma_s = W/A_s$ and $\sigma_d = F_d/A_d$, where subscripts 's' and 'd' represent the static and dynamic case, respectively. Furthermore, it is due to the fact that the change of the cross section is relatively small, the dynamic stress (σ_d), therefore, can be directly expressed as a function of dynamic factor (μ) and static stress (σ_s):

$$\sigma_d = \mu\sigma_s \ \ldots (19)$$

The static stress should be easily obtained from $\sigma_s = W/A_s$ and the dynamic factor from equations 4 and 16.

Once the maximum dynamic stress is calculated, it is possible to make an engineering decision for any problem under consideration, by comparing it with the allowable stress $[\sigma]$. In order to maintain the health and safety at the nuclear site, we at least must have the maximum dynamic stress to be less than or equal to the allowable stress, i.e.:

$$\sigma_{d\,\max} \le [\sigma] \ \ldots (20)$$

EXTENSION ACTIVITY – 1:

Draw a graph using a spreadsheet to illustrate the relationship between μ and h. Comment on your findings from this graph.

EXTENSION ACTIVITY – 2:

If the impactor is assumed to be an elastic body, how will it affect the dynamic factor μ? Whether it will be higher or less than that in case of a rigid impactor? What will happen if h is significantly greater than y_s?

WHERE TO FIND MORE

1. *Basic Engineering Mathematics*, John Bird, 2007, published by Elsevier Ltd.
2. *Engineering Mathematics*, Fifth Edition, John Bird, 2007, published by Elsevier Ltd.
3. *Strength of Materials*; S Timoshenko, 1976, published by Robert E. Krieger Publishing Company. INC.

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Jiansong obtained his PhD at Sheffield University, major on civil and structural engineering. He has worked in the UK industry for about 8 years after a number of years university lecturing and research. He is Chartered Engineer and a member of The Institute of Nuclear Engineer, and Institute of Mathematics and its Applications.

He says:

"*Mathematical models can be applied as an effective and efficient tool in engineering practice. It is essential for a professional engineer to possess a good mathematical grounding.*"

INFORMATION FOR TEACHERS

The teachers should have some knowledge of

- spring-mass system, Hooke's law
- dynamic and static stress and strain
- solving quadratic equations
- plotting graphs of quadratic functions

TOPICS COVERED FROM "MATHEMATICS FOR ENGINEERING"

- Topic 1: Mathematical Models in Engineering
- Topic 4: Functions

LEARNING OUTCOMES

- LO 01: Understand the idea of mathematical modelling
- LO 04: Understand the mathematical structure of a range of functions and be familiar with their graphs
- LO 09: Construct rigorous mathematical arguments and proofs in engineering context
- LO 10: Comprehend translations of common realistic engineering contexts into mathematics

ASSESSMENT CRITERIA

- AC 1.1: State assumptions made in establishing a specific mathematical model
- AC 1.2: Describe and use the modelling cycle
- AC 4.1: Identify and describe functions and their graphs
- AC 4.2: Analyse functions represented by polynomial equations
- AC 9.1: Use precise statements, logical deduction and inference
- AC 9.2: Manipulate mathematical expressions
- AC 9.3: Construct extended arguments to handle substantial problems
- AC 10.1: Read critically and comprehend longer mathematical arguments or examples of applications

LINKS TO OTHER UNITS OF THE ADVANCED DIPLOMA IN ENGINEERING

- Unit-1: Investigating Engineering Business and the Environment
- Unit-4: Instrumentation and Control Engineering
- Unit-5: Maintaining Engineering Plant, Equipment and Systems
- Unit-6: Investigating Modern Manufacturing Techniques used in Engineering
- Unit-7: Innovative Design and Enterprise
- Unit-8: Mathematical Techniques and Applications for Engineers
- Unit-9: Principles and Application of Engineering Science

ANSWERS TO EXTENSION ACTIVITIES

EA1: Use equation (16).

EA2: Consider equation (16) again to answer this question.